

4TH
SEMESTER

Waves and Optics

MARKS: 100

B.Sc. 2ND YEAR

4TH SEMESTER

2017-18

PHYSICS

~~UNIVERSITY OF KASHMIR~~

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UNITS	DESCRIPTION
UNIT-I	<i>The wave equation, general solution of one dimensional wave equation; Harmonic waves; Standing waves on a string of fixed length; Energy of a vibrating string The wave equation for longitudinal waves on a thin cylindrical rod; Energy density and energy transmission in waves; Application to Earthquakes. Standing waves on a stretched rectangular membrane: solution by method of separation of variables; normal modes of vibrations.</i>
UNIT-II	<i>General Theory of Image Formation: Cardinal Points of an optical system, general relationships, thick lens formula and lens combination, langrange equation of magnification. Abberations: Chromatic and monochromatic abberations and their reductions. Corrector plates.</i>
UNIT-III	<i>Review of Interference of light; Interference in thin parallel films; Application to Non-reflecting films, Newton rings; Michelson interferometer and its application for precision determination of wavelength; Multiple beam interference; Fabry-Perot interferometer and etalon; Intensity distribution. Fraunhofer diffraction at a slit; the intensity distribution; Two slit diffraction pattern; The intensity distribution.</i>
UNIT-IV	<i>Diffraction at N parallel slits; intensity distribution at an N parallel slits. Resolution of images; Rayleigh criterion; Resolving power of a diffraction grating. Frensel half-period zones; The Zone-Plate; Diffraction at a circular aperture; Diffraction by a straight edge (analysis using halfperiod zones). Polarization by reflection, Malus's law; Double refraction; Refraction in Uniaxial crystals; Optical activity; Rotation of plane of polarization; Origin of optical rotation in liquids and in crystals.</i>

UNIT-I

Q:- Define Waves & its characteristics.

A:- A stone is thrown on the still water of the pond. When the stone hits the water then the particles of water of that region start to vibrate. These vibrating particles vibrate the neighboring of particles. This the vibration reaches to the edge of the pond by vibrating the particles of water. The particles of water oscillate up and down but do not move forward. The periodic vibration that passes over the surface of water due to the oscillatory motion of the particles is called wave.

The mechanical energy produced due to the vibration of water particles is transferred from one place to another through vibration. So, energy is transferred from one place to another through wave.

The periodic motion of particles which transfers energy from one place to another through a material medium but does not displace the particles permanently is called wave. The wave produced in solid, liquid or gaseous medium is called mechanical wave. Water wave, sound wave etc. are mechanical waves. To transfer the mechanical wave elastic medium is needed. There is another type of wave that does not need any medium to propagate which are called electromagnetic wave.

The characteristics of waves are the followings:

1. The wave is produced due to the harmonic motion of the particles but the particles are not displaced permanently.
2. A medium is necessary for the propagation of mechanical wave.
3. The wave transfers energy from one place to another.
4. The velocity of wave depends on the nature of medium.
5. The reflection, refraction and superposition occur in case of wave.

Q:- The wave equation.

A:- The wave equation for a plane wave traveling in the x direction is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Where v is the phase velocity of the wave and y represents the variable which is changing as the wave passes. This is the form of the wave equation which applies to a stretched string or a plane electromagnetic. The mathematical description of a wave makes use of Partial derivatives.

In two dimensions, the wave equation takes the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$$

Which could describe a wave on a Stretched Membrane?

Waves in Ideal String

The wave equation for a wave in an ideal string can be obtained by applying Newton's 2nd Law to an infinitesimal segment of a string.

If a constant horizontal tension T is maintained in the string, then

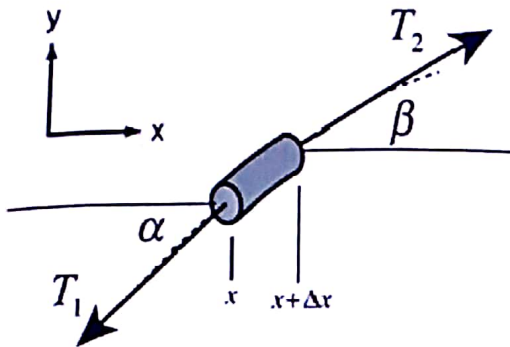
$$T_1 \cos \alpha = T_2 \cos \beta = T$$

Newton's 2nd law becomes

$$T_2 \sin \beta - T_1 \sin \alpha = \rho \Delta x \frac{\partial^2 y}{\partial t^2}$$

Combination of these two expressions for small angles gives

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$



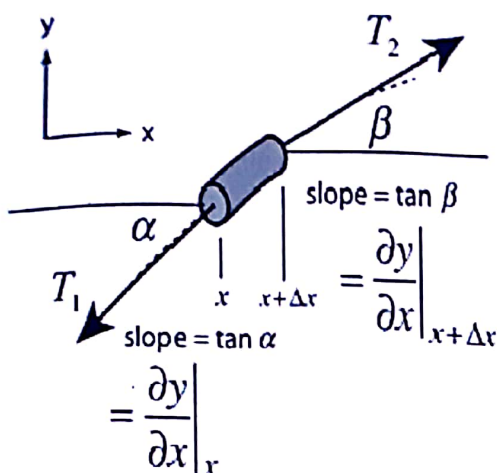
ρ = mass per unit length

String Wave Equation Development

$$T_1 \cos \alpha = T_2 \cos \beta = T$$

Analysis of the forces on a segment of string gives two relationships:

$$T_2 \sin \beta - T_1 \sin \alpha = \rho \Delta x \frac{\partial^2 y}{\partial t^2}$$



Combining gives

$$\frac{T_2 \sin \beta}{T_2 \cos \beta} - \frac{T_1 \sin \alpha}{T_1 \cos \alpha} = \frac{\rho \Delta x}{T} \frac{\partial^2 y}{\partial t^2}$$

and relating this to the slopes at the segment ends gives

$$\frac{1}{\Delta x} \left[\frac{\partial y}{\partial x} \Big|_{x+\Delta x} - \frac{\partial y}{\partial x} \Big|_x \right] = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$

In the limit $\Delta x \rightarrow 0$ this becomes

$$\rho = \text{mass per unit length} \quad \frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$

Q: - General solution of one dimensional wave equation.

A:- The wave equation is an important second-order linear partial differential equation for the description of waves – as they occur in physics – such as sound waves, light waves and water waves. It arises in fields like acoustics, electromagnetics, and fluid dynamics.

General solution

Algebraic approach

The one-dimensional wave equation is unusual for a partial differential equation in that a relatively simple general solution may be found. Defining new variables:

$$\xi = x - ct \quad ; \quad \eta = x + ct$$

Changes the wave equation into

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$$

Which leads to the general solution

$$u(\xi, \eta) = F(\xi) + G(\eta)$$

or equivalently:

$$u(x, t) = F(x - ct) + G(x + ct)$$

In other words, solutions of the 1D wave equation are sums of a right traveling function F and a left traveling function G . "Traveling" means that the shape of these individual arbitrary functions with respect to x stays

constant, however the functions are translated left and right with time at the speed c . This was derived by Jean le Rond d'Alembert.

Another way to arrive at this result is to note that the wave equation may be "factored":

$$\left[\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right] \left[\frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right] u = 0$$

and therefore:

$$\frac{\partial u}{\partial t} - c \frac{\partial u}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0$$

These last two equations are advection equations, one left traveling and one right, both with constant speed c . For an initial value problem, the arbitrary functions F and G can be determined to satisfy initial conditions:

$$\begin{aligned} u(x, 0) &= f(x) \\ u_t(x, 0) &= g(x) \end{aligned}$$

The result is d'Alembert's formula:

$$u(x, t) = \frac{f(x - ct) + f(x + ct)}{2} + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$$

In the classical sense if $f(x) \in C^k$ and $g(x) \in C^{k-1}$ then $u(t, x) \in C^k$. However, the waveforms F and G may also be generalized functions, such as the delta-function. In that case, the solution may be interpreted as an impulse that travels to the right or the left.

The basic wave equation is a linear differential equation and so it will adhere to the superposition principle. This means that the net displacement caused by two or more waves is the sum of the displacements which would have been caused by each wave individually. In addition, the behavior of a wave can be analyzed by breaking up the wave into components, e.g. the Fourier transform breaks up a wave into sinusoidal components.

Plane wave eigenmodes

Another way to solve for the solutions to the one-dimensional wave equation is to first analyze its frequency eigenmodes. A so-called eigenmode is a solution that oscillates in time with a well-defined angular frequency ω , with which the temporal part of the wave function for such eigenmode takes a specific form $e^{-i\omega t}$. The rest of the wave function is only dependent on the spatial variable x , hence amounting to separation of variables. Now writing the wave function as

$$u_\omega(x, t) = e^{-i\omega t} f(x)$$

We can obtain an ordinary differential equation for the spatial part $f(x)$

differential equation for the spatial

$$\frac{\partial^2 u_\omega}{\partial t^2} = \frac{\partial^2}{\partial t^2} [e^{-i\omega t} f(x)] = -\omega^2 e^{-i\omega t} f(x) = c^2 \frac{\partial^2}{\partial x^2} [e^{-i\omega t} f(x)]$$

$$\Rightarrow \frac{d^2}{dx^2} f(x) = -\left(\frac{\omega}{c}\right)^2 f(x)$$

Which is precisely an eigenvalue equation for $f(x)$, hence the name eigenmode. It has the well known plane wave solutions

$$f(x) = Ae^{\pm ikx},$$

With wave number

$$k = \omega/c.$$

The total wave function for this eigenmode is then the linear combination

$$u_{\omega}(x, t) = e^{-i\omega t} [Ae^{-ikx} + Be^{ikx}] = Ae^{-i(kx+\omega t)} + Be^{i(kx-\omega t)},$$

where complex numbers A, B depend in general on any initial and boundary conditions of the problem.

Eigenmodes are useful in constructing the full solution to the wave equation, because each of them evolves in time trivially with the phase factor $e^{-i\omega t}$. So with this, the full solution can always be decomposed into an eigenmode expansion

$$u(x, t) = \int_{-\infty}^{\infty} s(\omega) u_{\omega}(x, t) d\omega$$

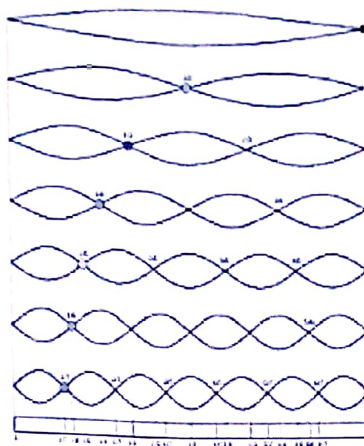
or in terms of the plane waves,

$$\begin{aligned} u(x, t) &= \int_{-\infty}^{\infty} s_{+}(\omega) e^{-i(kx+\omega t)} d\omega + \int_{-\infty}^{\infty} s_{-}(\omega) e^{i(kx-\omega t)} d\omega \\ &= \int_{-\infty}^{\infty} s_{+}(\omega) e^{-ik(x+ct)} d\omega + \int_{-\infty}^{\infty} s_{-}(\omega) e^{ik(x-ct)} d\omega \\ &= F(x-ct) + G(x+ct), \end{aligned}$$

Which is exactly in the same form as in the algebraic approach. Functions $s_{\pm}(\omega)$ are known as the Fourier component and are determined by initial and boundary conditions. This is a so-called frequency-domain method, alternative to direct time-domain propagations, such as FDTD method, of the wave packet $u(x, t)$.

Q: - Harmonic waves.

A: - Harmonic waves are described by sinusoidal functions. The wavelength is equal to linear distance between repetitions of transverse disturbance or phase. A **harmonic** of such a wave is a wave with a frequency that is a positive integer multiple of the frequency of the original wave, known as the fundamental frequency. The original wave is also called the 1st **harmonic**, the following **harmonics** are known as higher **harmonics**.

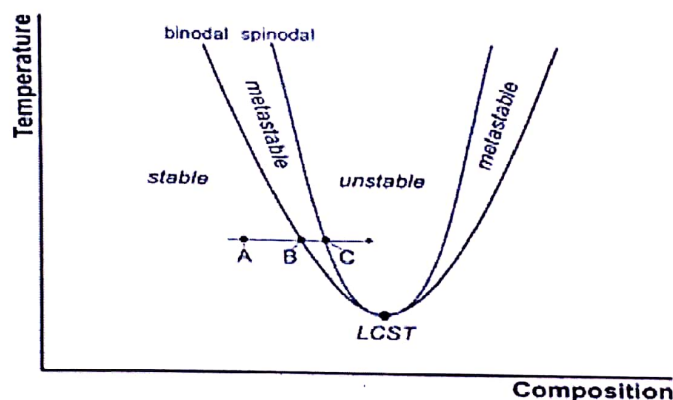


Simple harmonic motion can also be angular. In this case, the restoring torque required for producing SHM is directly proportional to the angular displacement and is directed towards the mean position.

Consider a wire suspended vertically from a rigid support. Let some weight be suspended from the lower end of the wire. When the wire is twisted through an angle θ from the mean position, a restoring torque acts on it tending to return it to the mean position. Here restoring torque is proportional to angular displacement θ .

Hence $\tau = -C\theta$

Where C is called torque constant.

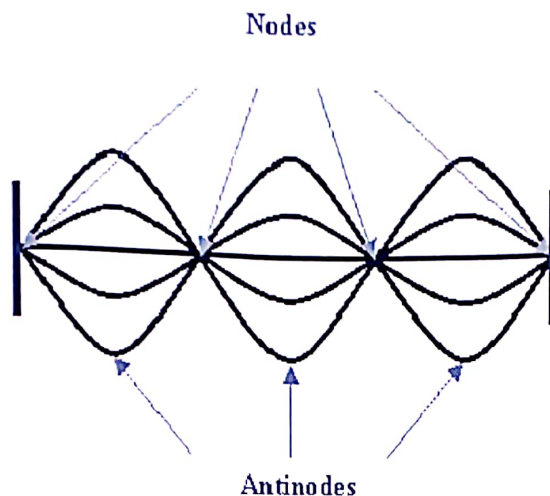


It is equal to the moment of the couple required to produce unit angular displacement. Its unit is N m rad^{-1} . The negative sign shows that torque is acting in the opposite direction to the angular displacement. This is the case of angular simple harmonic motion.

Examples: Tensional pendulum, balance wheel of a watch.

Q: - Standing waves on a string of fixed length

4:- A standing wave is a pattern which results from the interference of two or more waves traveling in the same medium. All standing waves are characterized by positions along the medium which are standing still. Such positions are referred to as nodes. Standing waves are also characterized by antinodes. These are positions along the medium where the particles oscillate about their equilibrium position with maximum amplitude. Standing wave patterns are always characterized by an alternating pattern of nodes and antinodes.



Transverse waves on a string

Standing waves of many different wavelengths can be produced on a string with two fixed ends, as long as an integral number of half wavelengths fits into the length of the string. For a standing wave on a string of length L with two fixed ends

$$L = n(\lambda/2), \quad n = 1, 2, 3, \dots$$

- Fundamental: $L = \lambda/2$, $n = 1$, $1/2$ wavelength fits into the length of the string.
- Second harmonic: $L = \lambda$, $n = 2$, one wavelength fits into the length of the string.
- Third harmonic: $L = 3\lambda/2$, $n = 3$, $3/2$ wavelengths fit into the length of the string.

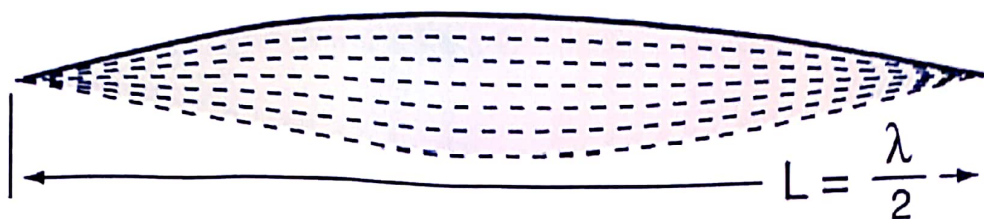
For a string the speed of the waves is a function of the mass per unit length $\mu = m/L$ of the string and the tension F in the string.

$$v = \sqrt{\frac{F}{\mu}}$$

Waves on a string with two fixed ends will be generated by a string vibrator. The waves will all have a frequency of 120 Hz. Their wavelength is given by $\lambda = v/f$. Since the frequency is fixed, the wavelength of the waves can only be changed by changing the speed of the waves. Students will adjust the tension in the string until 1, 2, or 3 half wavelength of a wave with $f = 120$ Hz fit into the length of the string. Then 120 Hz is a natural frequency of the string and the vibrator drives the string into resonance. The amplitude increases and the standing waves can easily be observed.

Q: - Vibrating String

A: - The fundamental vibrational mode of a stretched string is such that the wavelength is twice the length of the string.



Applying the basic wave relationship gives an expression for the fundamental frequency:

$$f_1 = \frac{v_{\text{wave on string}}}{2L} \quad \text{Calculation}$$

Since the wave velocity is given by $v = \sqrt{\frac{T}{m/L}}$, the frequency expression

can be put in the form:

$$f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}$$

T = string tension
 m = string mass
 L = string length

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing on the string.

Q: - Wave Velocity in String.

A: - The velocity of a traveling wave in a stretched string is determined by the tension and the mass per unit length of the string.

The wave velocity is given by $v = \sqrt{\frac{T}{m/L}}$

When the wave relationship is applied to a stretched string, it is seen that resonant standing wavemodes are produced. The lowest frequency mode for a stretched string is called the fundamental, and its frequency is given by

$$f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}$$

T = string tension

m = string mass

L = string length

From

Velocity = $\sqrt{\text{tension} / \text{mass per unit length}}$

Q: - Vibrating String Frequencies

A: - If you pluck your guitar string, you don't have to tell it what pitch to produce - it knows! That is, its pitch is its resonant frequency, which is determined by the length, mass, and tension of the string. The pitch varies in different ways with these different parameters, as illustrated by the examples below:

If you have a string with starting pitch:	100 Hz
and change* to	the pitch will be
double the length	50 Hz
four times the tension	200 Hz
four times the mass	50 Hz

If you want to raise the pitch of a string by increasing its tension:

	Tension	Frequency
Original	T_0	f_0
1 octave up	$4T_0$	$2f_0$
2 octaves up	$16T_0$	$4f_0$
3 octaves up	$64T_0$	$8f_0$
4 octaves up	$256T_0$	$16f_0$
5 octaves up	$1024T_0$	$32f_0$

*with the other parameters reset to their original values.

You can see that it is not practical to tune a string over a large pitch range using the tension, since the tension goes up by the square of the pitch ratio.

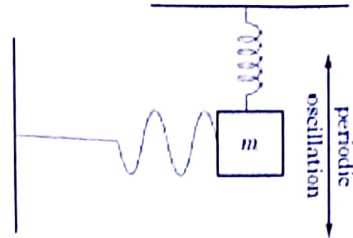
Q: - The wave equation for longitudinal waves on a thin cylindrical rod.

A: - There are two major kinds of waves: transverse waves and longitudinal waves. The medium transmitting transverse waves oscillates in a direction perpendicular to the direction the wave is traveling. A good example is waves on water: the water oscillates up and down while transmitting a wave horizontally. Other common examples include a wave on a string and electromagnetic waves. By contrast, the medium transmitting longitudinal waves oscillates in a direction parallel to the direction the wave is traveling. The most commonly discussed form of longitudinal waves is sound.

Transverse Waves: Waves on a String

Imagine—or better yet, go grab some twine and set up—a length of string stretched between two posts so that it is taut. Each point on the string is just like a mass on a spring: its equilibrium position lies on the straight line between the two posts, and if it is plucked away from its resting position, the string will exert a force to restore its equilibrium position, causing periodic oscillations. A string is more complicated than a simple mass on a spring, however, since the oscillation of each point influences nearby points along the string. Plucking a string at one end causes periodic vibrations that eventually travel down the whole length of the string. Now imagine detaching one end of the string from the pole and connecting it to a mass on a spring, which oscillates up and down, as in the figure below. The oscillation at one end of the string creates waves that propagate, or travel, down the length

of the string. These are called, appropriately, traveling waves. Don't let this name confuse you: the string itself only moves up and down, returning to its starting point once per cycle. The wave travels, but the medium—the string, in this case—only oscillates up and down.



The speed of a wave depends on the medium through which it is traveling. For a stretched string, the wave speed depends on the force of tension, F_T , exerted by the pole on the string, and on the mass density of the string, μ :

$$\mu = \frac{\text{total mass}}{\text{total length}}$$

The formula for the wave speed is:

$$v = \sqrt{\frac{F_T}{\mu}}$$

EXAMPLE



A string is tied to a pole at one end and 100 g mass at the other, and wound over a pulley. The string's mass is 100 g, and it is 2.5 m long. If the string is plucked, at what speed do the waves travel along the string? How could you make the waves travel faster? Assume the acceleration due to gravity is 10 m/s^2 .

Since the formula for the speed of a wave on a string is expressed in terms of the mass density of the string, we'll need to calculate the mass density before we can calculate the wave speed.

$$\mu = \frac{0.1 \text{ kg}}{2.5 \text{ m}} = 0.04 \text{ kg/m}$$

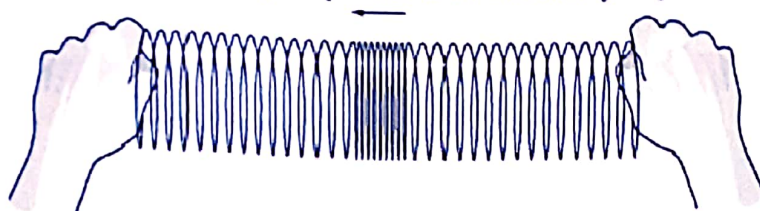
The tension in the string is the force of gravity pulling down on the weight, $F_T = mg = (0.1 \text{ kg})(10 \text{ m/s}^2) = 1 \text{ N}$. The equation for calculating the speed of a wave on a string is:

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{1 \text{ N}}{0.04 \text{ kg/m}}} = 5 \text{ m/s}$$

This equation suggests two ways to increase the speed of the waves: increase the tension by hanging a heavier mass from the end of the string, or replace the string with one that is less dense.

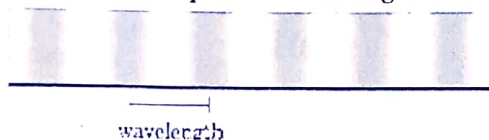
Longitudinal Waves: Sound

While waves on a string or in water are transverse, sound waves are longitudinal. The term longitudinal means that the medium transmitting the waves—air, in the case of sound waves—oscillates back and forth, parallel to the direction in which the wave is moving. This back-and-forth motion stands in contrast to the behavior of transverse waves, which oscillate up and down, perpendicular to the direction in which the wave is moving. Imagine a slinky. If you hold one end of the slinky in each of your outstretched arms and then jerk one arm slightly toward the other, you will send a pulse across the slinky toward the other arm. This pulse is transmitted by each coil of the slinky oscillating back and forth parallel to the direction of the pulse.



When the string on a violin, the surface of a bell, or the paper cone in a stereo speaker oscillates rapidly, it creates pulses of high air pressure, or compressions, with low pressure spaces in between, called rarefactions. These compressions and rarefactions are the equivalent of crests and troughs in transverse waves: the distance between two compressions or two rarefactions is a wavelength.

Pulses of high pressure propagate through the air much like the pulses of the slinky illustrated above, and when they reach our ears we perceive them as sound. Air acts as the medium for sound waves, just as string is the medium for waves of displacement on a string. The figure below is an approximation of sound waves in a flute—each dark area below indicates compression and represents something in the order of 10^{24} air molecules.



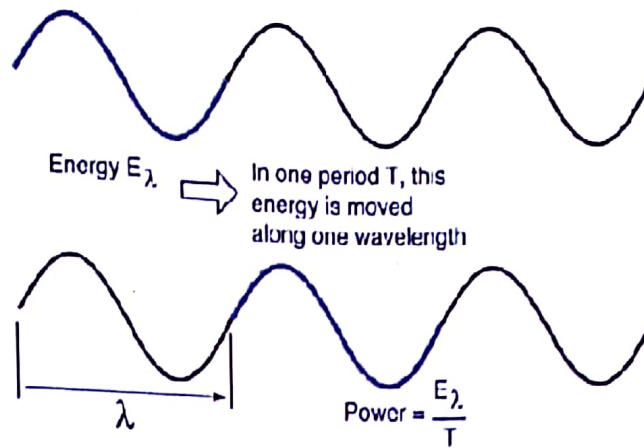
Loudness, Frequency, Wavelength, and Wave Speed

Many of the concepts describing waves are related to more familiar terms describing sound. For example, the square of the amplitude of a sound wave is called its loudness, or volume. Loudness is usually measured in decibels. The decibel is a peculiar unit measured on a logarithmic scale. You won't need to know how to calculate decibels, but it may be useful to know what they are.

The frequency of a sound wave is often called its pitch. Humans can hear sounds with frequencies as low as about 90 Hz and up to about 15,000 Hz, but many animals can hear sounds with much higher frequencies. The term wavelength remains the same for sound waves. Just as in a stretched string, sound waves in air travel at a certain speed. This speed is around 343 m/s under normal circumstances, but it varies with the temperature and pressure of the air. You don't need to memorize this number: if a question involving the speed of sound comes up on the SAT II, that quantity will be given to you.

Q: - How much power is transported by a string wave?

A: - As a sinusoidal wave moves down a string, the energy associated with one wavelength on the string is transported down the string at the propagation velocity v . From the basic wave relationship, the distance traveled in one period is $vT = \lambda$, so the energy is transported one wavelength per period of the oscillation.



The energy associated with one wavelength of the wave is

$$E_{\lambda} = \frac{1}{2} \mu \omega^2 A^2 \lambda$$

so the transported power is $= \frac{1}{2} \mu \omega^2 A^2 \frac{\lambda}{T} = \frac{1}{2} \mu \omega^2 A^2 v$

Power transmitted by string wave $= \frac{1}{2} \mu \omega^2 A^2 v$

Labels in the diagram:
 - Angular frequency of the wave (points to ω)
 - Wave propagation velocity (points to v)
 - Mass per unit length of the string (points to μ)
 - Wave amplitude (points to A)

Energy in a String Wave

The energy associated with a traveling wave in a stretched string is conveniently expressed as the energy per wavelength.

Potential energy $\Delta U = \frac{1}{2} (\Delta m) \omega^2 y^2$

Kinetic energy $\Delta KE = \frac{1}{2} \Delta m v_y^2$

Labels in the diagram:
 - A : Amplitude
 - y : Displacement
 - v_y : Transverse velocity
 - v : Wave propagation velocity

The energy of a small segment of the string can be expressed as the sum of the kinetic energy and elastic potential energy of the segment. The differential form of the elastic potential energy is

$$dU = \frac{1}{2} \omega^2 y^2 dm = \frac{1}{2} \omega^2 y^2 \mu dx$$

Using the description of a traveling wave

$$y = A \sin(kx - \omega t)$$

The potential energy expression becomes

$$dU = \frac{1}{2} \mu \omega^2 A^2 \sin^2(kx - \omega t) dx$$

The energy for a full wavelength can be found by integrating this expression at a given time, and it is most convenient to set $t=0$ for this integration. The energy for one wavelength along the string is

$$U_\lambda = \frac{1}{2} \mu \omega^2 A^2 \int_0^\lambda \sin^2(kx) dx$$

$$U_\lambda = \frac{1}{2} \mu \omega^2 A^2 \left[\frac{1}{2} x - \frac{1}{4k} \sin(2kx) \right]_0^\lambda = \frac{1}{4} \mu \omega^2 A^2 \lambda$$

The differential kinetic energy is

$$dKE = \frac{1}{2} v_y^2 dm = \frac{1}{2} v_y^2 \mu dx$$

Using the velocity expression

$$v_y = \omega A \cos(kx - \omega t)$$

the kinetic energy takes the form

$$dKE = \frac{1}{2} \mu \omega^2 A^2 \cos^2(kx - \omega t) dx$$

and again setting $t=0$ for simplification

$$KE_\lambda = \frac{1}{2} \mu \omega^2 A^2 \int_0^\lambda \cos^2(kx) dx = \frac{1}{4} \mu \omega^2 A^2 \lambda$$

The total energy associated with a wavelength is

$$E_{\lambda} = U_{\lambda} + KE_{\lambda} = \frac{1}{2} \mu \omega^2 A^2 \lambda$$

Since this amount of energy is transported a distance of one wavelength along the string in one period, this expression can be used to calculate the power transmitted along a string.

Q: - Energy density and energy transmission in waves.

A: - Wave represents distributed energy. Except for standing waves, the wave transports energy along with it from one point to another. In the context of our course, we shall focus our attention to the transverse harmonic wave along a string and investigate energy being transported by it. But the discussion and results would be applicable to other transverse waves as well through a medium. For a base case, we shall consider an ideal case in which there is no loss of energy.

We supply energy by continuously vibrating the free end of taut string. This energy is transmitted by the small vibrating string element to the neighboring element following it. We can see that vibrating small elements (also referred as particle) possess energy as it oscillates (simple harmonic motion for our consideration) in the transverse direction. It can be easily visualized as in the case of SHM that the energy has two components i.e. kinetic energy (arising from motion) and potential energy (arising from position). The potential energy is elastic potential energy like that of spring. The small string element is subjected to tension and thereby periodic elongation and contraction during the cycle of oscillatory motion.

There are few important highlights of energy transport. The most important ones are:

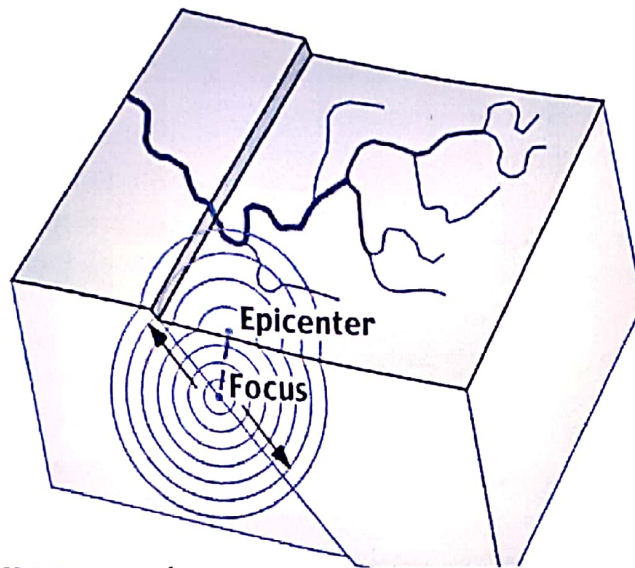
- The energy at the extreme positions along the string corresponds to zero energy.
- The kinetic and elastic potential energies at the mean (equilibrium) position are maximum.

Q: - Application to Earthquakes

A: - An earthquake is a sudden vibration or trembling in the Earth. More than 150,000 tremors strong enough to be felt by humans occur each year worldwide. Earthquake motion is caused by the quick release of stored potential energy into the kinetic energy of motion. Most earthquakes are produced along faults, tectonic plate boundary zones, or along the mid-oceanic ridges.

Earthquake Waves

Earthquakes are a form of wave energy that is transferred through bedrock. Motion is transmitted from the point of sudden energy release, the earthquake focus (hypocenter), as spherical seismic waves that travel in all directions outward (Figure 3). The point on the Earth's surface directly above the focus is termed the Epicenter. Two different types of seismic waves have been described by geologists: body waves and surface waves. Body waves are seismic waves that travel through the lithosphere. Two kinds of body waves exist: P-waves and S-waves. Both of these waves produce a sharp jolt or shaking. P-waves or primary waves are formed by the alternate expansion and contraction of bedrock and cause the volume of the material they travel through to change. They travel at a speed of about 5 to 7 kilometers per second through the lithosphere and about 8 kilometers per second in the asthenosphere. The speed of sound is about 0.30 kilometers per second. P-waves also have the ability to travel through solid, liquid, and gaseous materials. When some P-waves move from the ground to the lower atmosphere, the sound wave that is produced can sometimes be heard by humans and animals.



S-waves or secondary waves are a second type of body wave. These waves are slower than P-waves and can only move through solid materials. S-waves are produced by shear stresses and move the materials they pass through in a perpendicular (up and down or side to side) direction.

Surface waves travel at or near the Earth's surface. These waves produce a rolling or swaying motion causing the Earth's surface to behave like waves on the ocean. The velocity of these waves is slower than body waves. Despite their slow speed, these waves are particularly destructive to human construction because they cause considerable ground movement.

Let us describe the different types of waves in earthquakes.

1. **Body Waves:** They travel through the earth's interior and hence are called body waves. Ray paths are created by them as a result of refraction by the earth's surface that is of varying density and modulus. The density and modulus then vary in accordance to temperature, composition, and phase. The entire concept is very much similar to the refraction of light waves
2. **Primary Waves:** Primary waves are also called p waves. These are compressional waves. They are longitudinal in nature. These are a type of pressure waves. The speed of P waves is greater than other waves. These are called the primary waves as they are the first to arrive during the earthquake. This is because of large velocity. The propagation of these waves knows no bounds and hence can travel through any type of material, including fluids. They travel at nearly 2 times the speed of S waves.
3. **S-Wave:** S waves stand for shear waves. They resemble the jello bowl and vibrations in it. This is the way they travel. The second wave being felt during an earthquake is called a S wave. The speed of S wave is less than a P wave. The motion of S wave is restricted through solid rock only. The up and down, or side-to-side movement of the rocks is contributed to S waves. S waves motion is prohibited through a liquid. This is different from a p wave that can move through a liquid as it is a compression wave. It can be said that the rigidity of liquid is not enough for the transmission of S waves. The speed of S wave is slower than P wave and hence cannot move through a liquid. This information about the travel patterns of waves can be easily utilized by scientists to determine the structure of earth. S wave is generated by various reasons such as earthquakes,

volcano eruption, explosions, meteor hitting earth surface or by huge machines. It should be noted that the waves produced by earthquakes do not have a constant speed neither they travel in straight lines.

4. **Surface Wave:** These waves travel through the surface of the earth and hence are called the surface waves. They are also called L waves. They are similar to the waves in water. The speed of surface wave is less than body waves. This is the most dangerous and destructive type of destructive wave due to the long time period, larger amplitude, low frequency etc. the effect or strength of these waves diminish as they move away from the surface. Surface waves (L-waves) are analogous to water waves and travel along the Earth's surface. The speed of these waves is less than that of body waves. They are the most destructive wave. The name surface waves is common because they diminish as they get further from the surface.
5. **Rayleigh Waves:** These waves are also known as ground roll. These are also a type of surface wave. They travel as ripples on the surface. They are much similar to that of ripples on water. The restoring force here is elastic and not gravitational. The discovery of these waves was done by John William Strutt, Lord Rayleigh, in 1885. The speed of Rayleigh waves is less than body waves, roughly 90% of the velocity of S waves for typical homogeneous elastic media.
6. **Love Waves:** It is a horizontally polarized wave. the name is after A.E.H. Love. They have the largest amplitude.
7. **Stoneley Waves:** A Stoneley wave is a classification of Rayleigh wave with large amplitude.

Q: - Solution by method of separation of variables.

A: - The method of images and complex analysis are two rather elegant techniques for solving Poisson's equation. Unfortunately, they both have an extremely limited range of application. The final technique we shall discuss in this course, namely, the separation of variables, is somewhat messy, but possess a far wider range of application. Let us examine a specific example.

Consider two semi-infinite, grounded, conducting plates lying parallel to the x - z plane, one at $y = 0$, and the other at $y = \pi$ (see Fig. 46). The left end, at $x = 0$, is closed off by an infinite strip insulated from the two plates, and maintained at a specified potential $\phi_0(y)$. What is the potential in the region between the plates?

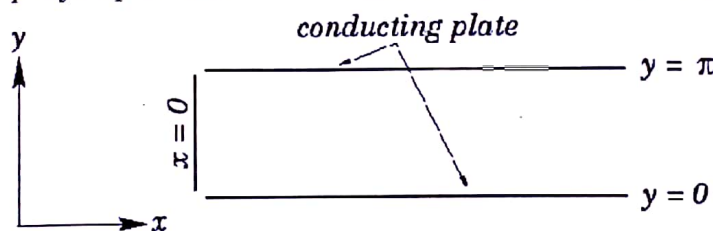


Figure 46:

We first of all assume that the potential is z -independent, since everything else in the problem is. This reduces the problem to two dimensions. Poisson's equation is written

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (771)$$

In the vacuum region between the conductors. The boundary conditions are

$$\phi(x, 0) = 0, \quad (772)$$

$$\phi(x, \pi) = 0 \quad (773)$$

for $x > 0$, since the two plates are earthed, plus

$$\phi(0, y) = \phi_0(y) \quad (774)$$

$$\text{for } 0 \leq y \leq \pi, \text{ and}$$

$$\phi(x, y) \rightarrow 0 \quad (775)$$

as $x \rightarrow \infty$. The latter boundary condition is our usual one for the scalar potential at infinity.

The central assumption in the method of separation of variables is that a multi-dimensional potential can be written as the product of one-dimensional potentials, so that

$$\phi(x, y) = X(x) Y(y). \quad (776)$$

The above solution is obviously a very special one, and is, therefore, only likely to satisfy a very small subset of possible boundary conditions. However, it turns out that by adding together lots of different solutions of this form we can match to general boundary conditions.

Substituting (776) into (771), we obtain

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} = 0. \quad (777)$$

Let us now separate the variables: i.e., let us collect all of the x -dependent terms on one side of the equation, and all of the y -dependent terms on the other side. Thus,

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -\frac{1}{Y} \frac{d^2 Y}{dy^2}. \quad (778)$$

This equation has the form

$$f(x) = g(y), \quad (779)$$

where f and g are general functions. The only way in which the above equation can be satisfied, for general x and y , is if both sides are equal to the same constant. Thus,

$$\frac{1}{X} \frac{d^2 X}{dx^2} = k^2 = -\frac{1}{Y} \frac{d^2 Y}{dy^2}. \quad (780)$$

The reason why we write k^2 , rather than $-k^2$, will become apparent later on. Equation (780) separates into two ordinary differential equations:

$$\frac{d^2 X}{dx^2} = k^2 X, \quad (781)$$

$$\frac{d^2 Y}{dy^2} = -k^2 Y. \quad (782)$$

We know the general solution to these equations:

$$X = A \exp(kx) + B \exp(-kx), \quad (783)$$

$$Y = C \sin(ky) + D \cos(ky), \quad (784)$$

Giving

$$\phi = [A \exp(kx) + B \exp(-kx)] [C \sin(ky) + D \cos(ky)]. \quad (785)$$

Here, A , B , C , and D are arbitrary constants. The boundary condition (775) is automatically satisfied if $A = 0$ and $k > 0$. Note that the choice k^2 , instead of $-k^2$, in Eq. (780) facilitates this by making ϕ either grow or decay monotonically in the x -direction instead of oscillating. The boundary condition (772) is automatically satisfied if $D = 0$. The boundary condition (773) is satisfied provided that

$$\sin(k\pi) = 0, \quad (786)$$

which implies that k is a positive integer, n (say). So, our solution reduces to

$$\phi(x, y) = C \exp(-nx) \sin(ny), \quad (787)$$

where B has been absorbed into C . Note that this solution is only able to satisfy the final boundary condition (774) provided $\phi_0(y)$ is proportional to $\sin(ny)$. Thus, at first sight, it would appear that the method of separation of variables only works for a very special subset of boundary conditions. However, this is not the case.

Now comes the clever bit! Since Poisson's equation is linear, any linear combination of solutions is also a solution. We can therefore form a more general solution than (787) by adding together lots of solutions involving different values of n . Thus,

$$\phi(x, y) = \sum_{n=1}^{\infty} C_n \exp(-n x) \sin(n y), \quad (788)$$

where the C_n are constants. This solution automatically satisfies the boundary conditions (772), (773) and (775). The final boundary condition (774) reduces to

$$\phi(0, y) = \sum_{n=1}^{\infty} C_n \sin(n y) = \phi_0(y). \quad (789)$$

The question now is what choice of the C_n fits an arbitrary function $\phi_0(y)$? To answer this question we can make use of two very useful properties of the functions $\sin(ny)$. Namely, that they are mutually orthogonal, and form a complete set. The orthogonality property of these functions manifests itself through the relation

$$\int_0^{\pi} \sin(n y) \sin(n' y) dy = \frac{\pi}{2} \delta_{nn'}, \quad (790)$$

where the function $\delta_{nn'} = 1$ if $n = n'$ and 0 otherwise is called a Kroenecker delta. The completeness property of sine functions means that any general function $\phi_0(y)$ can always be adequately represented as a weighted sum of sine functions with various different n values. Multiplying both sides of Eq. (789) by

$\sin(n' y)$, and integrating over y , we obtain

$$\sum_{n=1}^{\infty} C_n \int_0^{\pi} \sin(n y) \sin(n' y) dy = \int_0^{\pi} \phi_0(y) \sin(n' y) dy. \quad (791)$$

The orthogonality relation yields

$$\frac{\pi}{2} \sum_{n=1}^{\infty} C_n \delta_{nn'} = \frac{\pi}{2} C_{n'} = \int_0^{\pi} \phi_0(y) \sin(n' y) dy, \quad (792)$$

$$C_n = \frac{2}{\pi} \int_0^\pi \phi_0(y) \sin(ny) dy. \quad (793)$$

Thus, we now have a general solution to the problem for any driving potential $\phi_0(y)$.

If the potential $\phi_0(y)$ is constant then

$$C_n = \frac{2\phi_0}{\pi} \int_0^\pi \sin(ny) dy = \frac{2\phi_0}{n\pi} [1 - \cos(n\pi)], \quad (794)$$

giving

$$C_n = 0 \quad (795)$$

for even n , and

$$C_n = \frac{4\phi_0}{n\pi} \quad (796)$$

for odd n . Thus,

$$\phi(x, y) = \frac{4\phi_0}{\pi} \sum_{n=1,3,5} \frac{\exp(-nx) \sin(nx)}{n}. \quad (797)$$

In the above problem, we write the potential as the product of one-dimensional functions. Some of these functions grow and decay monotonically (i.e., the exponential functions), and the others oscillate (i.e., the sinusoidal functions). The success of the method depends crucially on the orthogonality and completeness of the oscillatory

functions. A set of functions $f_n(x)$ is orthogonal if the integral of the product of two different members of the set over some range is always zero: i.e.,

$$\int_a^b f_n(x) f_m(x) dx = 0, \quad (798)$$

for $n \neq m$. A set of functions is complete if any other function can be expanded as a weighted sum of them. It turns out that the scheme set out above can be generalized to more complicated geometries. For instance, in

spherical geometry, the monotonic functions are power law functions of the radial variable, and the oscillatory functions are Legendre polynomials. The latter are both mutually orthogonal and form a complete set. There are also cylindrical, ellipsoidal, hyperbolic, toroidal, etc. coordinates. In all cases, the associated oscillating functions are mutually orthogonal and form a complete set. This implies that the method of separation of variables is of quite general applicability.

Finally, as a simple example of the solution of Poisson's equation in spherical geometry, let us consider the case of a conducting sphere of radius a , centered on the origin, placed in a uniform z -directed electric field

of magnitude E_0 . The scalar potential ϕ satisfies $\nabla^2 \phi = 0$ for $r \geq a$, with the boundary conditions $\phi \rightarrow -E_0 r \cos \theta$ (giving $\mathbf{E} \rightarrow E_0 \hat{z}$) as $r \rightarrow \infty$, and $\phi = 0$ at $r = a$. Here, r and θ are spherical polar coordinates. Let us try the simplified separable solution

$$\phi(r, \theta) = r^m \cos \theta. \quad (799)$$

It is easily demonstrated that the above solution satisfies $\nabla^2 \phi = 0$ provided $m = 1$ or -2 . Thus, the most general solution of $\nabla^2 \phi$ which satisfies the boundary condition at $r \rightarrow \infty$ is

$$\phi(r, \theta) = -E_0 r \cos \theta + \alpha r^{-2} \cos \theta. \quad (800)$$

The boundary condition at $r = a$ is satisfied provided

$$\alpha = E_0 a^3. \quad (801)$$

Of course, $\phi = 0$ inside the sphere (i.e., $r < a$). The charge sheet density induced on the surface of the sphere is given by

$$\sigma = \epsilon_0 E_r(r = a) = -\epsilon_0 \frac{\partial \phi(r = a)}{\partial r} = 3 \epsilon_0 E_0 \cos \theta.$$

Q: - Normal modes of vibrations.

A: - A mode of vibration is characterized by a modal frequency and a mode shape. It is numbered according to the number of half waves in the vibration. For example, if a vibrating beam with both ends pinned displayed a mode shape of half of a sine wave (one peak on the vibrating beam) it would be vibrating in mode 1. If it had a full sine wave (one peak and one valley) it would be vibrating in mode 2.

In a system with two or more dimensions, such as the pictured disk, each dimension is given a mode number. Using polar coordinates, we have a radial coordinate and an angular coordinate. If you measured from the center outward along the radial coordinate you would encounter a full wave, so the mode number in the radial direction is 2. The other direction is trickier, because only half of the disk is considered due to the antisymmetric (also called skew-symmetry nature of a disk's vibration in the angular direction. Thus, measuring

180° along the angular direction you would encounter a half wave, so the mode number in the angular direction is 1. So the mode number of the system is 2-1 or 1-2, depending on which coordinate is considered the "first" and which is considered the "second" coordinate (so it is important to always indicate which mode number matches with each coordinate direction).

Each mode is entirely independent of all other modes. Thus all modes have different frequencies (with lower modes having lower frequencies) and different mode shapes.

UNIT-II

Lenses

A lens is a piece of transparent material that can focus a transmitted beam of light. This is usually bounded by two spherical surfaces, or a spherical and a plane surface.

□> Explanation <□

A piece of transparent substance (commonly glass, plastic, quartz etc.) bounded by two surfaces of regular curvature is known as "Lens". A lens is used to change the curvature of wavefronts so that light may be focused to a desired position.

Basically thin lenses fall into two categories:

Converging or Convex

Diverging or Concave

A convex lens is thicker in the middle and thinner at the edges, converges light rays towards its optical axis, so that a beam of parallel rays converges at a point F (shown in figure given below). For example in bright sun light, a convex lens may produce a spot of light intense enough to ignite paper.

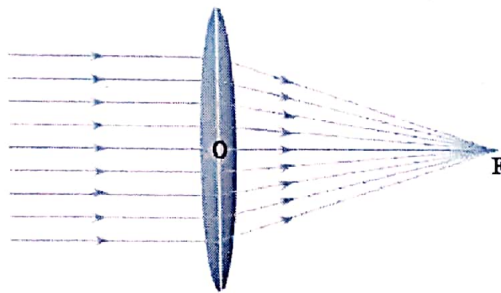


Fig Convex lens and its Principal Focus or Focal Point

A concave lens which is thinner in the middle and thicker at the edges bends rays outward from its optical axis (see figure below)

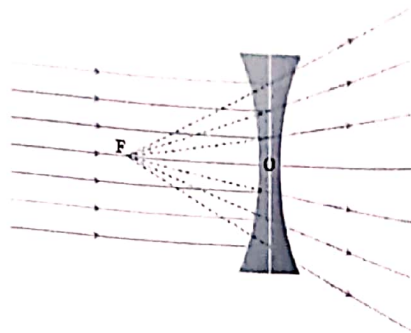
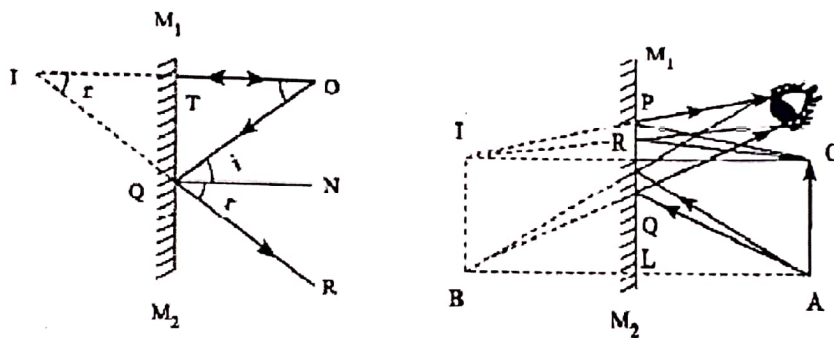


Fig Concave lens and its Principal Focus or Focal Point

The point F to which the rays are brought to focus is called the **principal focus** or the **focal point**, and the distance between the optical center of the lens and its principal focus is called the **focal length f** . Conventionally f is taken to be **positive** for convex lenses and **negative** for concave lenses.

Q:-General Theory of Image Formation

A:-The most general optical system is considered, in which the object and image may also be curved surfaces. For each object point a base ray is chosen, whose intersection with the image surface defines the geometrical image point. It is shown that, with suitable choices of axes for the object and image spaces, principal azimuths always exist around the object space and image space parts of the base ray.



These azimuths are such that, in a paraxial type approximation, rays entering with coordinates (XS, O) and (O, yT) , respectively, emerge with coordinates $(X'S, O)$ and $(O, y'T)$. This theorem enables the definition of canonical entrance and exit pupil variables, (xS, yT) and $(x'S, y'T)$, and object and image space variables, (GS, HT) and $(G'S, H'T)$, such that the geometrical image of (GS, HT) is at $G'S = GS, H'T = HT$, and (for an isoplanatic image point) any finite aperture ray entering with coordinates (xS, yT) emerges with $x'S = xS, y'T = yT$. The analysis also leads to formulas for the two principal local magnifications of the image and to the sine condition for a general optical system. Both the geometrical and diffraction theory of image formation for a general optical system are in this manner shown to reduce to exactly the same forms as for an axially symmetric system. In Part 2, the necessary computing methods are obtained for the practical application of the theory.

Cardinal Points of an Optical System

the points on the optical axis OO' (see Figure 1) of a centered optical system that can be used to construct the image of an arbitrary point in space for objects in the paraxial region, which is the region around the axis of symmetry of the system where a point is represented by a point, a straight line by a straight line, and a plane by a plane.

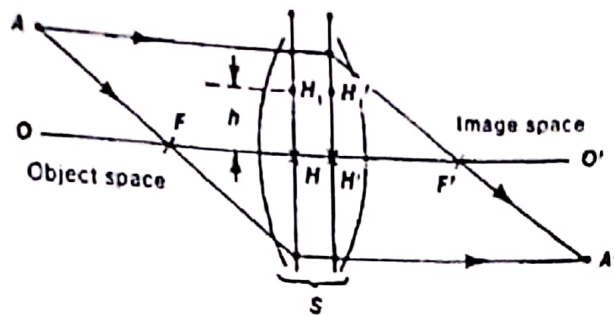


Figure 1. Position of the image A' of an arbitrary point A projected by an optical system S can be found if the cardinal points F , F' , H , and H' of the system are known: a ray passing through the front focus F is directed by the system parallel to its optical axis OO' , and a ray that is incident upon the system parallel to the axis OO' is directed through the back focus F' after refraction.

There are four cardinal points in an optical system: the front and back foci F and F' and the front and back principal points H and H' . The back focus is the image of an infinitely remote point located on the optical axis in the object space, and the front focus is the image in the object space of an infinitely remote point in the image space. The principal points are the points of intersection with the optical axis of the principal planes, which are the planes for which the optical system S produces full

size mutual images (every point H_1 located in the principal plane HH_1 at a distance h from the axis OO' appears in the other principal plane $H'H'_1$ as the point H'_1 at the same distance h from the axis as point H_1).

The distance from point H to point F is called the front focal distance (negative in the figure), and the distance from point H' to point F' is called the back focal distance (positive in the figure).

The construction of an image A' of an arbitrary point A for a centered optical system using the points F , H , H' , and F' is shown in Figure 1.

The Thin Lens Formula

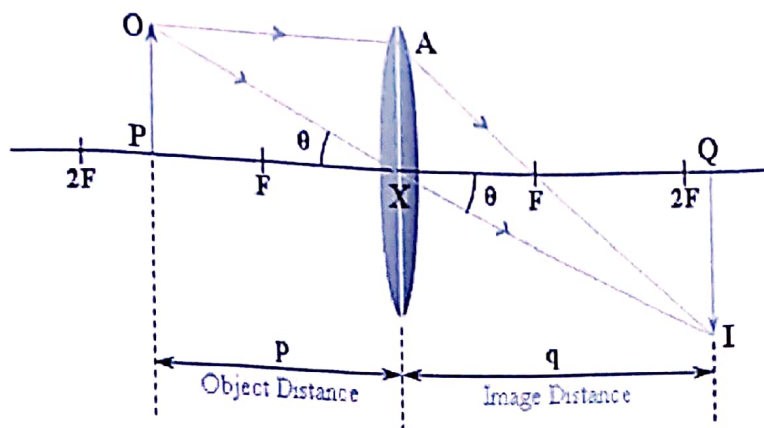
The nature and position of an image formed by a lens can be calculated from the lens formula.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Here p is the distance of object from the lens and q is the distance of image from the lens and f is the focal length of the lens.

Explanation

Let us consider an object OP whose real but inverted image is formed by a thin convex lens as shown in the figure:



It is clear from the figure that $\triangle OXP$ and $\triangle IXQ$ are similar then according to theorem of geometry,

$$\frac{OP}{PX} = \frac{IQ}{QX}$$

$$\Rightarrow \frac{OP}{IQ} = \frac{PX}{QX}$$

$$\frac{OP}{IQ} = \frac{p}{q} \quad \Rightarrow (1)$$

In figure, $\triangle AFX$ and $\triangle IFQ$ are also similar then according to geometry,

$$\frac{AX}{FX} = \frac{IQ}{FQ}$$

$$\frac{AX}{IQ} = \frac{FX}{FQ} \quad \because AX = OP$$

$$\frac{OP}{IQ} = \frac{FX}{QX - FX}$$

$$\frac{OP}{IQ} = \frac{f}{q - f} \quad \Rightarrow (2)$$

Comparing equation (1) and (2),

$$\Rightarrow \frac{q - f}{q} = \frac{f}{p}$$

$$\Rightarrow 1 - \frac{f}{q} = \frac{f}{p}$$

$$\Rightarrow 1 = \frac{f}{p} + \frac{f}{q}$$

Or,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

This equation is known as the lens formula.

Combination of Thin Lenses

The image of an object can be made erect and much magnified by using more than one lenses. Therefore suitable combination of lenses are used in high powered optical instruments.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The above expression is for the combination of two lenses where f_1 is the focal length of one lens and f_2 is the focal length of the other.

⇒ Explanation <□

In most of the optical instruments two or more lenses are used in combination. The location, size and nature of the final image can be determined by using lens formula or ray diagram. In either case, we locate first the image formed by the first lens. Using that image as the object for the second lens, the final image formed by the second lens can be located. If there are more than two lenses. This process is continued, the object for each lens is the image for the preceding lens as in the figure below:

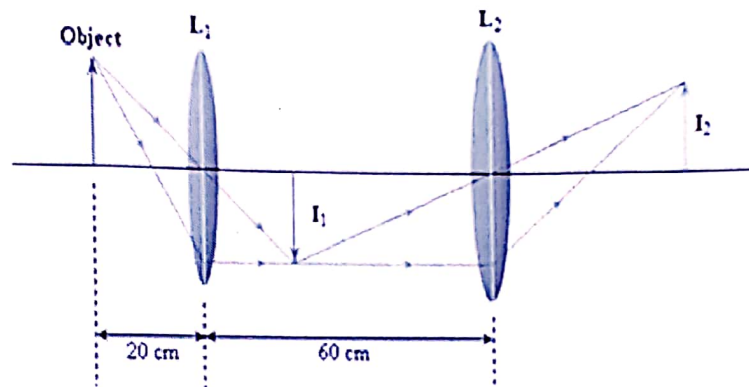
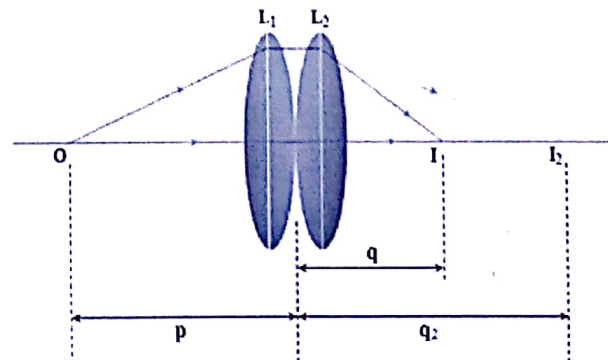


Fig Showing a combination of thin lenses

Referring to the above figure, we can see that lens L_1 forms an image I_1 . This image acts as a real object for the lens L_2 , which forms a real image I_2 . Notice that I_2 is inverted with respect to I_1 and erect with respect to the object.

We now consider the case when the two thin lenses are in contact, which means their separation is very small as compared to their focal lengths. This is illustrated in figure below:



Let a point object O be placed at a distance p from the lens L_1 whose real image I_1 (in fig 12) is formed by it at a distance q_1 (in fig 12). From the lens formula,

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1} \Rightarrow (1)$$

Where f_1 is the focal length of lens L_1 .

This image now serves as a virtual object for the second lens L_2 of focal length f_2 . If we neglect the small separation between the lenses. The distance of this virtual object from lens L_2 will be the same as its distance from the lens L_1 . If the lens forms an image I of this virtual object at a distance q .

$$-\frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2} \Rightarrow (2)$$

As the object is virtual for lens L_2 i.e. $P_2 = q_1$,

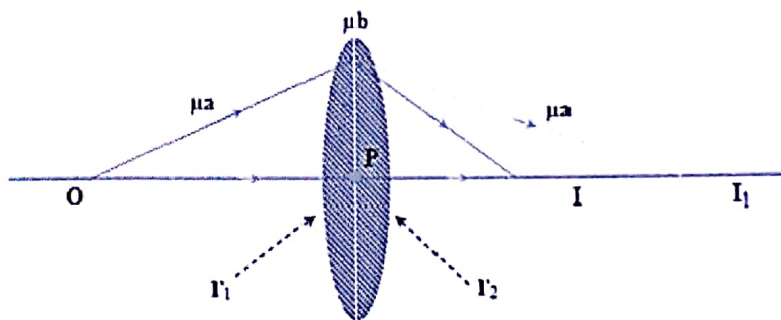
Adding equation (1) and (2) we get,

$$\frac{1}{p} + \frac{1}{q_1} - \frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$$

Or,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow (3)$$

Now if we replace the two lenses of focal lengths f_1 and f_2 by single lens of focal length f such that it forms an image at a distance q of an object placed at p from it as shown in figure below, such a lens is called **equivalent lens**, and its focal length is known as **equivalent focal length**.



For equivalent lens L , we have

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow (4)$$

Comparing equation (3) and (4) we get,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow (5)$$

This close combination behaves as a single lens whose focal length is given by the above relation.

The equation (5) shows that for a pair of lenses in contact the sum of the reciprocals of their individual focal lengths is equal to the reciprocal of the focal length of the combination.

Compound Microscope and its Magnification

It is an optical device used to observe very small objects like bacteria, cells, gas molecules and section of plants etc.

> Explanation < 1

Whenever high magnification is desired, a compound microscope is used. It consists of two convex lenses, one placed before the object called Objective and other placed before the eye called Eye-piece.

The objective has very small focal length f_o and small aperture, whereas eye-piece has relatively large focal length f_e and large aperture.

Working

When a small object OP is placed at distance p just before the focus of objective then a real, inverted and magnified image IQ is formed at distance q which falls within the focus of eye-piece. The eye-piece is used as magnifying glass and it takes up the image IQ as an object and since it lies within the focal length of eye-piece then it forms a virtual and much magnified final image $I'Q'$ as shown below.

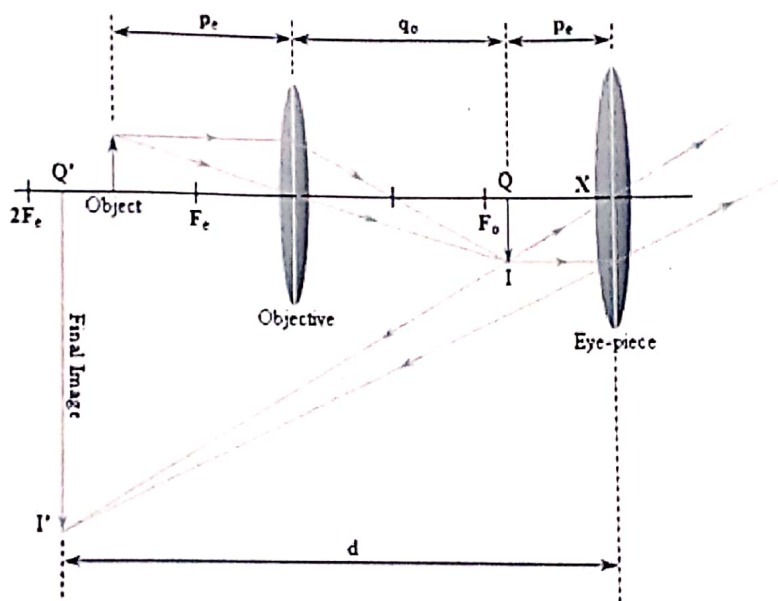


Fig Ray diagram of Compound Microscope

Magnification

If α and β are the visual angles subtended by the object and image respectively when formed at the least distance of distinct vision. Then magnifying power of microscope will be

$$M = \frac{\beta}{\alpha} \Rightarrow (1)$$

where $\alpha = \frac{OP}{d}$, when object OP is placed at least distance of distinct vision in front of eye. $\tan \alpha \approx \alpha$ for small angle.
And in right angle triangle $\Delta Q'XI'$

$$\tan \beta = \frac{I'Q'}{XQ}$$

$$\beta = \frac{I'Q'}{d}$$

Now putting the values of α and β in equation (1) we get,

$$\Rightarrow M = \frac{I'Q'}{d} \times \frac{d}{OP}$$

$$\Rightarrow M = \frac{I'Q'}{OP}$$

It can be written as,

$$M = \frac{I'Q'}{IQ} \times \frac{IQ}{OP}$$

$$M = M_2 \times M_1 \Rightarrow (2)$$

Where

M_1 is the magnification of objective

M_2 is the magnification of eye-piece

As,

$$M_1 = \frac{q_o}{p_o}$$

And since eye-piece is used as magnifying glass then,

$$M_2 = 1 + \frac{d}{f_e}$$

Therefore equation (2) can be written as,

$$\Rightarrow M = \frac{q_o}{p_o} \left(1 + \frac{d}{f_e} \right) \Rightarrow (3)$$

This expression gives the magnification of eye-piece.

As the object is placed very close to the focus of objective then,

$$p_o \approx f_o$$

And since IQ is formed very close to the eye-piece then,

$$q_o \approx L$$

Where L is the distance between objective and eye-piece and it is also known as length of the microscope. Now equation (3) may be written as,

$$M \approx \frac{L}{f_o} \left(1 + \frac{d}{f_e} \right)$$

The limit to which a microscope can be used to resolve details, depends on the width of the objective. A wider objective and use of blue light of short wavelength produces less diffraction and allows more details to be seen.

Q:- Simple Microscope or Magnifying Glass and its Magnification

A:- An optical instrument, which is used to see small objects distinctly is called Simple Microscope or Magnifying glass.

Explanation $\angle$

A convex lens which help us to observe clearly an object (very small object) which is brought close to eye i.e. within least distance of distinct vision, is called Magnifying Glass.

When the object is placed inside the focal point of the lens, then erect, magnified and virtual image is formed at near point or 25 cm.

The angular magnification of the magnifying glass is defined as the ratio of visual angle subtended by the image seen through a magnifying glass to the visual angle subtended by the object when placed at the least distance of distance of distinct vision and seen through naked eye i.e.

$$M = \frac{\beta}{\alpha} \Rightarrow (1)$$

Now suppose a small object OP forms a visual angle " α ", when it is placed at least distance of vision and when it is placed at distance " p " within focal length of magnifying glass then its erect, virtual and magnified image IQ is formed at least distance of vision and it subtends a visual angle " β " as shown in the figures below:

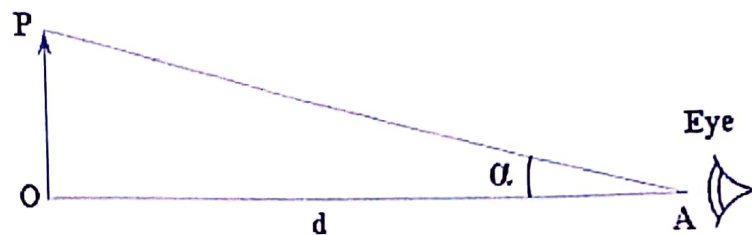


Fig A Object Angle

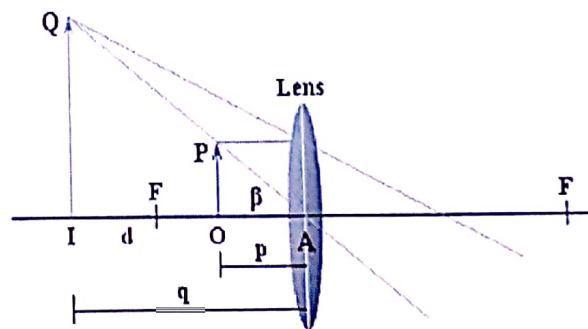


Fig B Image Angle

In figure A, the right angle triangle $\triangle OAP$ gives,

$$\tan \alpha = \frac{OP}{OA}$$

For small angle $\tan \alpha \approx \alpha$

$$\alpha = \frac{OP}{d}$$

from figure B, the right angle triangle ΔOAP gives,

$$\tan \beta = \frac{OP}{OA}$$

$$\beta = \frac{OP}{p}$$

Putting the values of α and β in equation (1), we get

$$M = \frac{OP}{p} \times \frac{d}{OP}$$

or,

$$M = \frac{d}{p}$$

According to the lens formula,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

As the final image is virtual then,

$$q = -d$$

$$\Rightarrow \frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiplying both sides by "d",

$$\Rightarrow \frac{d}{f} = \frac{d}{p} - \frac{d}{d}$$

since $M = \frac{d}{p}$,

$$\Rightarrow \frac{d}{f} = M - 1$$

or,

$$M = 1 + \frac{d}{f}$$

This expression gives magnifying power of magnifying glass or simple microscope.

It is thus obvious that for a lens of high angular magnification the focal length should be small. If, for example, $f = 5$ cm and $d = 25$ cm, then $M = 5$, the object would look five times larger when viewed through such a lens.

Power and Aberration of Lenses

The reciprocal of focal length of lens is called Power of lens i.e.

$$\text{power of lens} = \frac{1}{\text{focal len}} = \frac{1}{f}$$

The defect (Blurring and distortion) in the image formed by lenses is due to two common reasons i.e. spherical aberration and chromatic aberration. These are known as Lens Aberrations or defects of lens.

→ Explanation →

The reciprocal of focal length of a lens is its power. The greater the focal length, the lesser will be the power of lens and smaller the focal length, the greater will be the power of lens.
In case of combination of two lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow P = P_1 + P_2$$

This shows that the power of combination of two lenses in contact is equal to the sum of their individual powers.

Unit

The unit of power of lens is "Diopter" and it is defined as the reciprocal of focal length in meters.
i.e.

$$1 \text{ Diopter (D)} = \frac{1}{1 \text{ meter}}$$

Defects of Lenses or Lens Aberrations

1-Spherical Aberration

When rays of light parallel to the principal axis of a lens pass through zones near its edges, these are brought to focus nearer the lens than those rays which pass through region nearer to its center. Therefore, the focus is not a sharp point but is contained in a region called "focal region". This produces blurred images. This problem is known as spherical aberration as shown in figure:

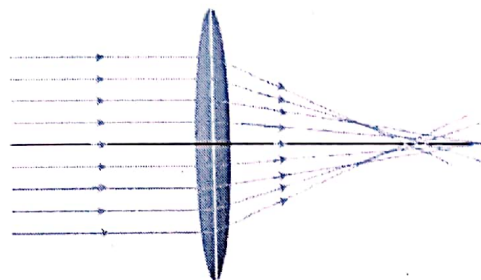


Fig Spherical Aberration

Minimizing

- By using only central portion of the lens. This can be achieved by using a stop on the lens which makes effective aperture of the lens small.
- By making opposite surface of the lens of different curvatures.
- By combining a strongly convergent lens of producing little spherical aberration in the opposite direction.

Chromatic Aberration

The fact that different wavelengths of light refracted by lens focus at different points give rise to chromatic aberration as shown in figure:

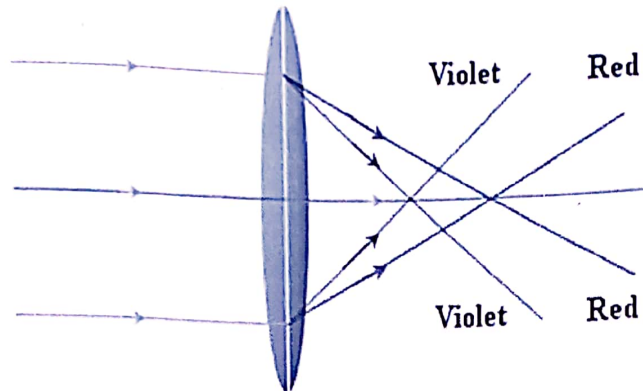


Fig Chromatic Aberration

Actually a thick lens may be regarded as made up of two prisms placed one above the other and due to dispersion of the refracting medium of the lens different wavelength of light focus at different points. Hence image formed by lens consists of small linear spectrum. Chromatic aberration for a diverging lens is opposite to that for a converging lens.

Minimizing

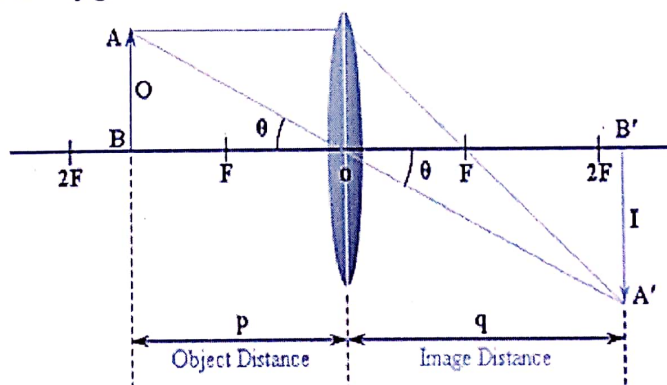
- This defect can be reduced to a large extent by combining a converging lens of crown glass with a diverging lens of flint glass. These lenses are so chosen that the dispersion of one is equal and opposite to that the other. Lenses which are free from dispersion are called "Achromatic lenses".

Q: - Resolving Power.

A:-The resolving power of an instrument is its ability to reveal the minor details of the object under examination.

□> Explanation <□

When an object is placed in front of a convex lens at a point beyond its focus, a real and inverted image of the object is formed as shown in the figure below:



The ratio of the size of the image to the size of the object is called magnification. As the object is brought from a far off point to the focus, the magnification goes on increasing. The apparent size of the an object depends on the angle subtended by it at the eyes. Thus, the closer the object is to the eye, the greater is the angle subtended and larger appears the size of the object. The maximum size of an object as seen by naked eye is obtained when the object is placed at the least distance of distinct vision. For lesser distance, the image formed looks blurred and the details of the object are not visible. The magnifying power or angular magnification is defined as the ratio of the angles subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.

The optical resolution of a microscope or a telescope tells us how close together the two point sources of light can be so that they are still seen as two separate sources. If two point sources are too close, they will appear as one because the optical instrument makes a points source look like a small disc or spot of light with circular diffraction fringes.

Although the magnification can be made as large as one desires by choosing appropriate focal lengths, but the magnification alone is of no use unless we can see the details of the object distinctly.

The resolving power of an instrument is its ability to reveal the minor details of the object under examination.

Resolving Power is expressed as the minimum angle which two point sources subtended at the instrument so that their images are seen two distinct spots of light rather than one. Raleigh showed that for light of wavelength λ through a lens of diameter D , the resolving power is given by

$$\alpha_{\min} = 1.22 \frac{\lambda}{D}$$

In the case of a grating spectrometer, the resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

Where $\lambda \approx \lambda_1 \approx \lambda_2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Thus, we see that a grating with high resolving power can distinguish small difference in wavelength. If N is the number of rulings on the grating, it can be shown that the resolving power in the m th-order diffraction equals to the product $N \times m$, i.e.

$$R = N \times m$$

Q:-Least Distance of Distinct Vision

A:-The minimum distance from the eye at which an object appears to be distinct is called the least distance of distinct vision or near point.

□> Explanation <□

The normal human eye can focus a sharp image of an object on the eye if the object located anywhere from infinity to a certain point called the "Near Point".

The minimum distance of an object from eye to have its clear image is called "Least Distance of Distinct Vision".

This distance is about 25 cm from the eye. It is denoted by d . If the object is held closer to the eye than this distance the image formed will be blurred and fuzzy. The location of the near point, however, changes with age.

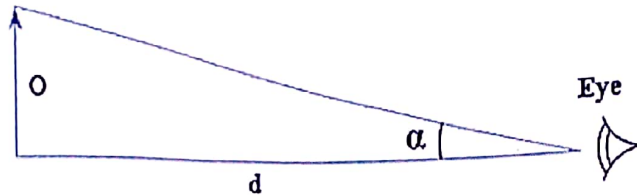
Visual Angle

The angle subtended by an object at the retina of eye is called "Visual Angle". The apparent size of an object depends upon its actual size and on the angle subtended by it at the eye. Thus the closer the object is to the eye, the greater is the angle subtended and larger appears the size of the object.

The greater the visual angle, the greater is the apparent size of the object.

Object Angle

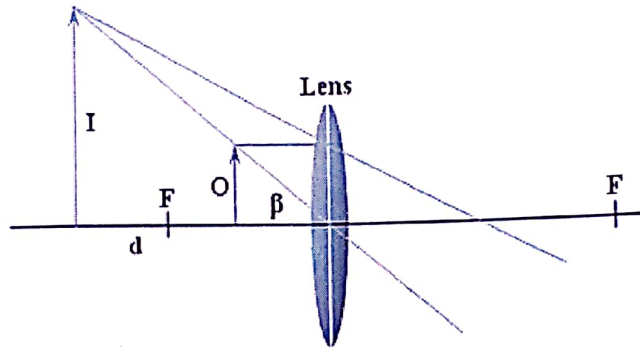
When the object is displaced at d , then the angle subtended at a naked (unaided) eye, by the object is called the "Object Angle". It is denoted by α (alpha).



Ray diagram for Object Angle

Image Angle

When the angle subtended by the image, formed at d , on the naked eye, is called "Image angle". It is denoted by β (beta).



Ray diagram for Image Angle

Linear Magnification

The ratio of the size of the image to the size of the object is called "Magnification". It is denoted by M .

$$M = \frac{I}{O} = \frac{q}{p}$$

Where,

I = Image Size

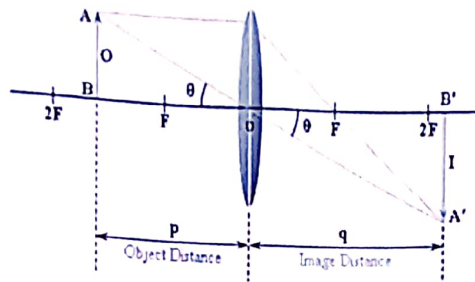
O = Object Size

p = Distance of object from the lens or mirror

q = Distance of image from the lens or mirror

Derivation for the Above Equation

When an object is placed in front of a convex lens at a point beyond its focus, a real and inverted image of the object is formed as shown in the figure:



From the above figure, $\triangle ABO$ and $\triangle A'B'O$ are similar triangles.

For similar triangles the ratio of the length of the corresponding sides is equal i.e.,

$$\frac{I}{O} = \frac{OB'}{OB}$$

Since

$$OB' = q = \text{Image Distance}$$

$$OB = p = \text{Object Distance}$$

Therefore,

$$\frac{I}{O} = \frac{q}{p}$$

As,

$$M = \frac{I}{O}$$

Therefore,

$$M = \frac{I}{O} = \frac{q}{p}$$

Angular Magnification

The ratio of the angles subtended by the image as seen through the optical device (aided eye) to that subtended by the object at the naked (unaided) eye, is called "**Angular Magnification**". It is also called the magnifying power, Mathematically,

$$M = \frac{\beta}{\alpha}$$

UNIT-III

Q: - Review of Interference of light

A:-Interference of light:-Two slits A and B illuminated by a single monochromatic source S act as coherent sources. The waves from these two coherent sources travel in the same medium and superpose at various points as shown in Figure. The crest of the wavetrains are shown by thick continuous lines and troughs are shown by broken lines. At points where the crest of one wave meets the crest of the other wave or the trough of one wave meets the trough of the other wave, the waves are in phase, the displacement is maximum and these points appear bright. These points are marked by crosses (x). This type of interference is said to be constructive interference.

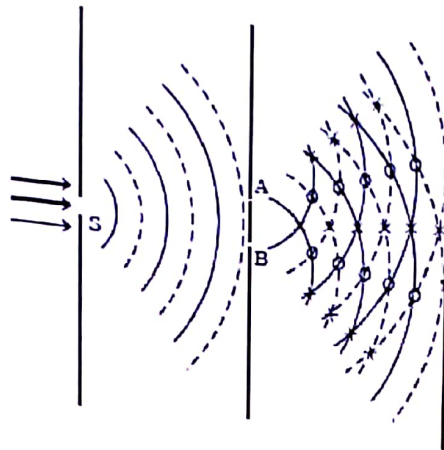


Fig: Interference phenomenon

At points where the crest of one wave meets the trough of the other wave, the waves are in opposite phase, the displacement is minimum and these points appear dark. These points are marked by circles (O). This type of interference is said to be destructive interference. Therefore, on a screen XY the intensity of light will be alternatively maximum and minimum i.e. bright and dark bands which are referred as interference fringes. The redistribution of intensity of light on account of the superposition of two waves is called interference.

The intensity of light (I) at a point due to a wave of amplitude (a) is given by $I \propto a^2$.

If a_1 and a_2 are the amplitude of the two interfering waves, then

$$I_1 \propto a_1^2 \text{ and } I_2 \propto a_2^2$$

$$\text{so, } I_1/I_2 = a_1^2/a_2^2$$

For constructive interference, $I_{\max} \propto (a_1 + a_2)^2$ and for destructive interference, $I_{\min} \propto (a_1 - a_2)^2$

$$I_{\max}/I_{\min} = (a_1 + a_2)^2/(a_1 - a_2)^2$$

Q: - Interference in thin parallel films

A: - Consider a transparent thin film of uniform thickness t and its refractive index μ bounded by two plane surfaces K and K' (Figure).

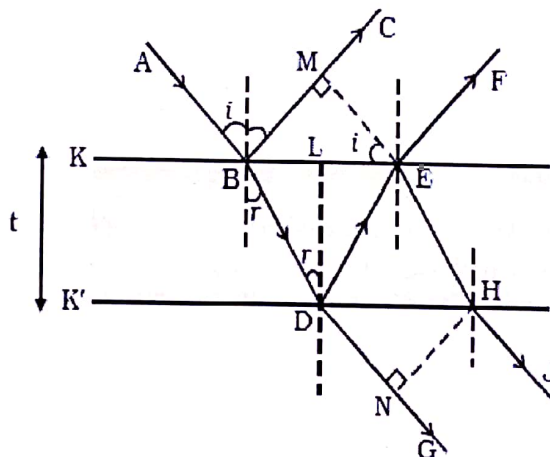


Fig: Interference in thin films

A ray of monochromatic light AB incident on the surface K of the film is partly reflected along BC and partly refracted into the film along BD. At the point D on the surface K', the ray of light is partly reflected along DE and partly transmitted out of the film along DG. The reflected light then emerges into air along EF which is parallel to BC. The ray EH after refraction at H, finally emerges along HJ.

Q: - Application to Non-reflecting films

A: - An antireflective or anti-reflection (AR) coating is a type of optical coating applied to the surface of lenses and other optical devices to reduce reflection. This improves the efficiency of the system since less light is lost. In complex systems such as a telescope, the reduction in reflections also improves the contrast of the image by elimination of stray light. This is especially important in planetary astronomy. In other applications, the primary benefit is the elimination of the reflection itself, such as a coating on eyeglass lenses that makes the eyes of the wearer more visible to others, or a coating to reduce the glint from a covert viewer's binoculars or telescopic sight.

Applications

Anti-reflective coatings are used in a wide variety of applications where light passes through an optical surface, and low loss or low reflection is desired. Examples include anti-glare coatings on corrective lenses and camera lens elements.

Corrective lenses

Opticians dispense "anti-reflection lenses" because the decreased reflection makes them look better, and they produce less glare, which is particularly noticeable when driving at night or working in front of a computer monitor. The decreased glare means that wearers often find their eyes are less tired, particularly at the end of the day. Allowing more light to pass through the lens also increases contrast and therefore increases visual acuity.

Antireflective ophthalmic lenses should not be confused with polarized lenses, which decrease (by absorption) the visible glare of sun reflected off surfaces such as sand, water, and roads. The term "antireflective" relates to the reflection from the surface of the lens itself, not the origin of the light that reaches the lens.

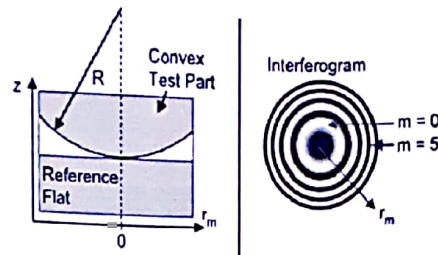
Many anti-reflection lenses include an additional coating that repels water and grease, making them easier to keep clean. Anti-reflection coatings are particularly suited to high-index lenses, as these reflect more light without the coating than a lower-index lens (a consequence of the Fresnel equations). It is also generally easier and cheaper to coat high index glasses.

Photolithography

Antireflective coatings are often in microelectronic photolithography to help reduce image distortions associated with reflections off the surface of the substrate. Different types of antireflective coatings are applied either before or after the photoresist, and help reduce standing waves, thin-film interference, and specular reflections.

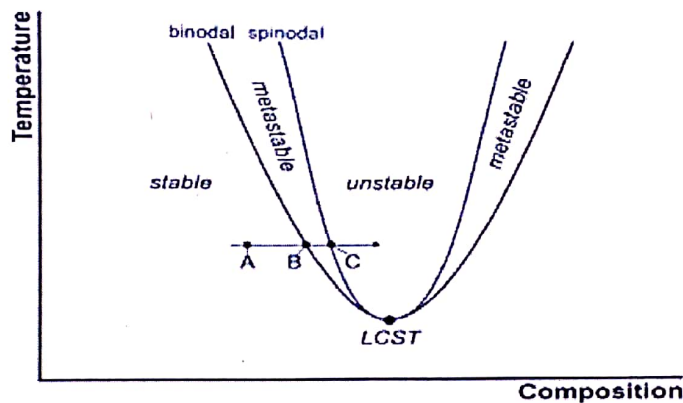
Q: - Explain Newton's Rings Theory

∴ The formation of Newton's rings can be explained on the basis of interference between waves which are normally reflected from the top and bottom surfaces of the air film. If t is the thickness of the air film at a point on the film, the refracted wavelet from the lens has to travel a distance t into the film and after reflection from the top surface of the glass plate, has to travel the same distance back to reach the point again.



$$R = \frac{r_m^2}{\lambda \left(m + \frac{1}{2} \right)}$$

Thus, it travels a total path $2t$. One of the two reflections takes place at the surface of the denser medium and hence it introduces an additional phase change of π or an equivalent path difference $\lambda/2$ between two wavelets.



∴ The condition for brightness is,
Path difference, $\delta = 2t + \lambda/2 = n\lambda$

$$2t = (2n-1) \lambda/2$$

Where $n = 1, 2, 3 \dots$ and λ is the wavelength of light used.

The condition for darkness is, path difference $\delta = 2t + \lambda/2 = (2n+1) \lambda/2$

$$2t = n\lambda$$

Where $n = 0, 1, 2, 3 \dots$

The thickness of the air film at the point of contact of lens L with glass plate P is zero. Hence, there is no path difference between the interfering waves. So, it should appear bright. But the wave reflected from the denser glass plate has suffered a phase change of π while the wave reflected at the spherical surface of the lens has not suffered any phase change. Hence the point O appears dark. Around the point of contact alternate bright and dark rings are formed.

Q:-Define & describe on Newton's Rings.

An important application of interference in thin films is the formation of Newton's rings. When a plano convex lens of long focal length is placed over an optically plane glass plate, a thin air film with varying thickness is enclosed between them.

The thickness of the air film is zero at the point of contact and gradually increases outwards from the point of contact. When the air film is illuminated by monochromatic light normally, alternate bright and dark concentric circular rings are formed with dark spot at the centre. These rings are known as Newton's rings.

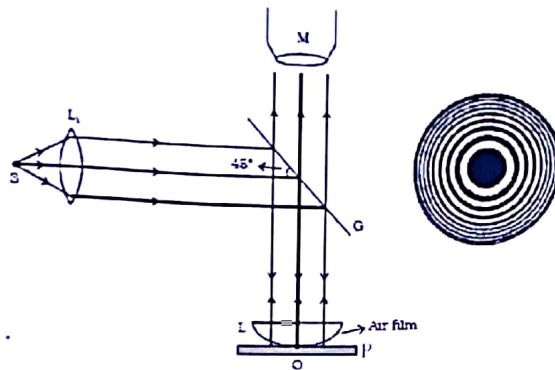


Fig: Newton's rings

Q:-Describe Applications of Newton's rings

(i) Using the method of Newton's rings, the wavelength of a given monochromatic source of light can be determined. The radius of n th dark ring and $(n+m)$ th dark ring are given by

$$r_n^2 = nR\lambda \text{ and } r_{n+m}^2 = (n+m)R\lambda$$

$$r_{n+m}^2 - r_n^2 = mR\lambda$$

$$\text{So, } \lambda = [r_{n+m}^2 - r_n^2] / mR$$

Knowing r_{n+m} , r_n and R , the wavelength can be calculated.

(ii) Using Newton's rings, the refractive index of a liquid can be calculated. Let λ_a and λ_m represent the wavelength of light in air and in medium (liquid). If r_n is the radius of the n th dark ring in air and if r_n' is the radius of the n th dark ring in liquid, then

$$r_n^2 = nR\lambda_a$$

$$r_n'^2 = nR\lambda_m = nR\lambda_a/\mu$$

$$\text{so, } \mu = r_n^2 / r_n'^2$$

Experiment of Newton's Rings

Figure shows an experimental arrangement for producing and observing Newton's rings. A monochromatic source of light is kept at the focus of a condensing lens. The parallel beam of light emerging from lens falls on the glass plate kept at 45° . The glass plate reflects a part of the incident light vertically downwards, normally on the thin air film, enclosed by the plano convex lens and plane glass plate.

The reflected beam from the air film is viewed with a microscope. Alternate bright and dark circular rings with dark spot as centre is seen.

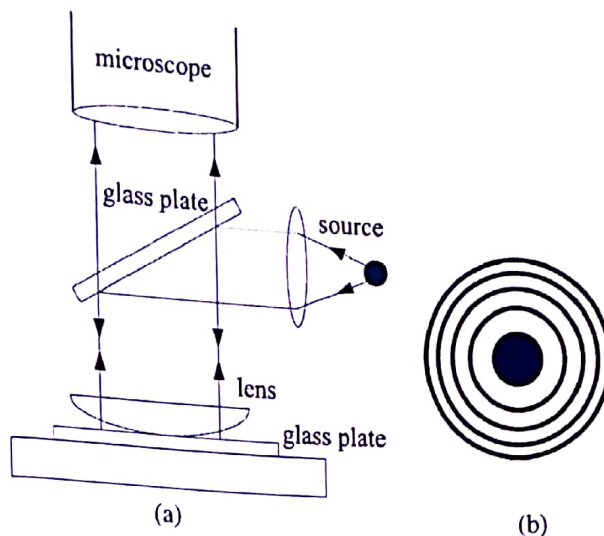
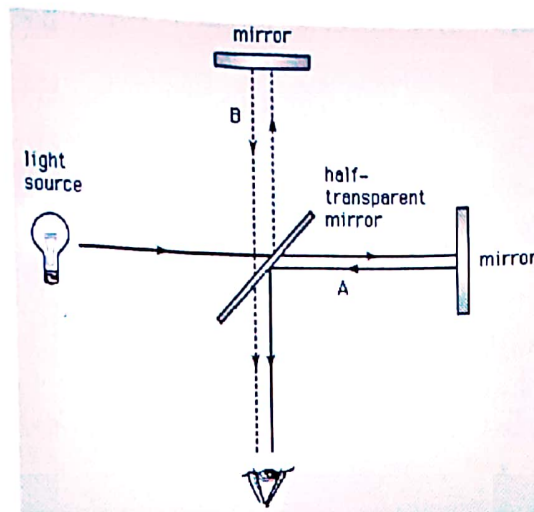


Fig: Newton's rings

Q: - Michelson interferometer and its application for precision determination of wavelength. Multiple beam interference; Fabry-Perot interferometer and etalon.

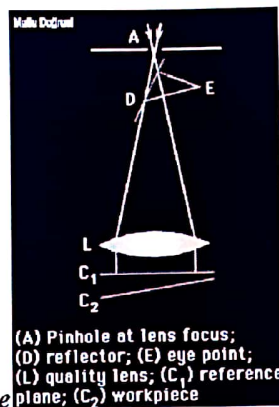
A: - An interferometer is an instrument for making precise measurements for beams of light of such factors as length, surface irregularities, and index of refraction. It divides a beam of light into a number of beams that travel unequal paths and whose intensities, when reunited, add or subtract (interfere with each other). This interference appears as a pattern of light and dark bands called interference fringes. Information derived from fringe measurements is used for precise wavelength determinations, measurement of very small distances and thicknesses, the study of spectrum lines, and determination of refractive indices of transparent materials. In astronomy, interferometers are used to measure the distances between stars and the diameters of stars.

In 1881 the American physicist A.A. Michelson constructed the interferometer used in the Michelson-Morley experiment.(see figure) The Michelson interferometer and its modifications are used in the optical industry for testing lenses and prisms, for measuring index of refraction, and for examining minute details of surfaces (micro topographies). The instrument consists of a half-silvered mirror that divides a light beam into two equal parts, one of which is transmitted to a fixed mirror and the other of which is reflected to a movable mirror. By counting the fringes created as the mirror is moved, the amount of movement can be precisely determined. Michelson also developed the stellar interferometer, capable of measuring the diameters of stars in terms of the angle, as small as $0.01''$ of an arc, subtended by the extreme points of the star at the point of observation.



In 1896 the British physicist Lord Rayleigh described the Rayleigh interference refractometer, still widely used for determining the refractive indices of gases and liquids. It is a split-beam instrument, like the Michelson interferometer. One beam serves as a reference, while the other is passed first through a material of known index of refraction and then through the unknown. The index of refraction of the unknown can be determined by the displacement of its interference fringes from those of the known material.

The Fabry interferometer (variable-gap interferometer) was produced in 1897 by the French physicists Charle Fabry. It consists of two highly reflective and strictly parallel plates called an etalon. Because of the high reflectivity of the plates of the etalon, the successive multiple reflections of light waves diminish very slowly in intensity and form very narrow, sharp fringes. These may be used to reveal hyperfine structures in line spectra, to evaluate the widths of narrow spectral lines, and to redetermine the length of the standard meter.



Fizeau-Laurent

surface

interferometry system:-The Fizeau-Laurent

surface interferometer reveals departures of polished surfaces from a plane. The system was described by the French physicist A.-H.-L. Fizeau in 1862 and adapted in 1883 into the instruments now widely used in the optical industry. In the Fizeau-Laurent system, monochromatic light (light of a single color) is passed through a pinhole and illuminates a reference plane and a work piece directly below it. The light beam is perpendicular to the work piece. By maintaining a slight angle between the surface of the work piece and the surface of the plane of reference, fringes of equal thickness can be seen through a reflector placed above them. The fringes constitute a

...our map of the surface of the work piece, enabling an optical polisher to see and to remove defects and departures from flatness.

The Twyman-Green interferometer, an adaptation of the Michelson instrument introduced in 1916 by the English electrical engineer Frank Twyman and the English chemist Arthur Green, is used for testing lenses and prisms. It uses a point source of monochromatic light at the focus of a quality lens. When the light is directed toward a perfect prism, it returns to a viewing point exactly as it was from the source, and a uniform field of illumination is seen. Local imperfections in the prism glass distort the wave front. When the light is directed toward a lens backed by a convex mirror, it passes through the lens, strikes the mirror, and retraces its path through the lens to a viewing point. Imperfections in the lens result in fringe distortions.

Multiple Beam Interference

If the two inner surfaces of the plates shown in Figure are coated so as to make them reflect 80 percent or more of the incident light, then the resulting interference pattern will be caused by the superposition of many beams.

The below figure shows an arrangement for producing the fringes of constant inclination by multiple-beam interference. The amplitudes of successive beams are proportional to r , r squared, r cubed, etc. (r is the ratio of the intensity of the reflected light to that of the incident light for one reflection). The phase differences are ϵ , 2ϵ , 3ϵ , etc., in which

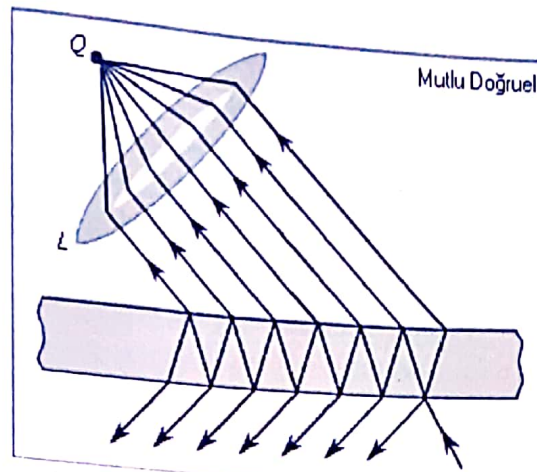
$$\epsilon = (4\pi e \cos\theta)/\lambda.$$

These fringes are much sharper than those obtained with two-beam interference. With a large number of beams the intensity is extremely high when they are all in phase ($\epsilon = 0$), but, even when the phase difference between any two successive beams (e.g., the first and the second) is quite small, the phase difference between the first and, say, the thirtieth beam is so large that the later beams in the series are in opposition to the earlier beams. Thus the intensity is relatively small except when the value of ϵ is close to one of the values $2\pi p$ (in which p is an integer). Multiple-beam fringes of constant inclination were used by Charles Fabry and Alfred Pérot in France for resolution of spectral lines having only small differences of wavelength. Multiple-beam fringes of constant thickness have been used by an English physicist, Samuel Tolansky, to detect surface irregularities down to less than a nanometre.

The Fabry-Perot Interferometer and Etalon

Until the development of all-dielectric multi-layer reflection coatings, which have low light-loss coefficients, the Fabry - Perot interferometer had very little application to the spectrometry of faint light sources, although it was used by Fabry and Buisson as early as 1911 on the relatively bright Orion nebula, though with lossy metallic layers.

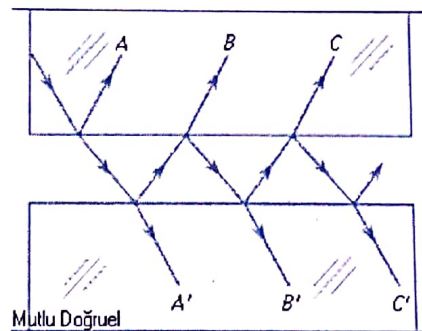
The most common form that it has taken is with two plane parallel reflecting layers. However, there is an interesting spherical version which is not widely used. Here the more common version will be emphasised:



The Fabry-Pérot interferometer consists of two reflecting mirrors that can be either curved or flat. In the Fabry-Pérot interferometer the separation between two semi-silvered glass plates can be varied. One plate remains stationary with respect to the frame of the instrument whilst the other is mounted on a nut threaded on an accurate screw.

The adjustment of the Fabry -Perot interferometer is in many ways similar to that of Michelson. In the Fabry - Perot interferometer, the multiple beam interference fringes from a plane parallel plate illuminated near normal incidence are used. The inner surfaces are coated with partially transparent films of high reflectivity and are parallel, so that they enclose a plane parallel plate of air. The plates themselves are made slightly prismatic, in order to avoid disturbing effects due to reflections at the outer uncoated surfaces.

Only certain wavelengths of light will resonate in the cavity: the light is in resonance with the interferometer if and only if $m(\lambda/2) = L$, where L is the distance between the two mirrors, m is an integer, and λ is the wavelength of the light inside the cavity. When this condition is fulfilled, light at these specific wavelengths will build up inside the cavity and be transmitted out the back end for specific wavelengths. By adjusting the spacing between the two mirrors, the instrument can be scanned over the spectral range of interest.



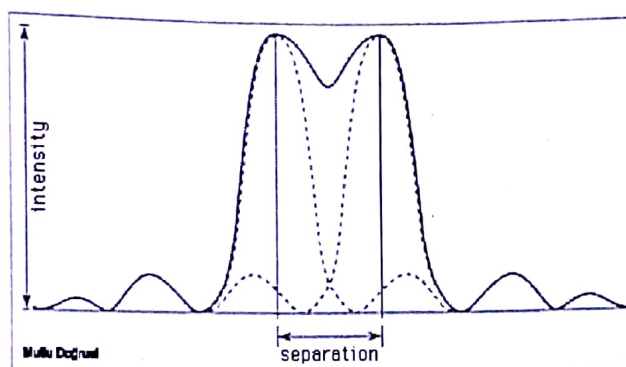
Consider a narrow, monochromatic beam from an extended source point making an angle (in air) of θ with respect to the optical axis of the system. The single beam produces multiple coherent beams in the interferometer, and the emerging set of parallel rays are brought together at some point P in the focal plane of the converging lens L . The nature of the superposition at P is determined by the path difference between successive parallel beams; taking the refraction index for air as 1, the condition for brightness is

$$2t \cos \theta = m \lambda.$$

Other beams from different points of the source but in the same plane and making the same angle θ with the axis satisfy the same path difference and also arrive at P. With t fixed, the above equation is satisfied for certain angles θ , and the fringe system is the familiar concentric rings due to the focusing of fringes of equal inclination. When collimating lens is used between source and interferometer, every set of parallel beams entering the etalon must arise from the same source point.

Resolution

When two wavelength components are present in the incident light, the Fabry-Perot interferometer gives a double set of circular fringes, each set belonging to one of the wavelengths. Such a plot for two wavelength components of comparable irradiance is shown in the figure. Although the two peaks are shown separately (the dashed lines), only the sum of the two, which follows the solid line enveloping the overall shape, is measured. Clearly, if the wavelengths are very close, also the fringes are close, and it may not be possible to distinguish two separate peaks in the measured irradiance. The minimum wavelength separation that can be resolved by the instrument depends on one's ability to detect the dip in the measured pattern between peaks.



Applications Of Fabry-Perot Interferometers

The Fabry-Perot interferometers have a wide range of applications. All these applications, however, are either based on the study of the fine structure lines, or the comparison of wavelengths as discussed under the title "resolution", above. When a Fabry-Perot interferometer is illuminated by some quasi-monochromatic light, the intensity distribution of the transmitted light differs from its standard form and yields some information about the spectral distribution of the light used. If we imagine that their wavelength difference is gradually increased, and providing they do not differ too greatly in intensity, their presence will eventually be evident from the presence of two mutually displaced sets of maxima in the interference pattern. The components are then said to be resolved by the interferometer. In this way, Fabry and Perot were able to observe directly the fine structure of spectral lines which Michelson could only infer, and the Fabry-Perot interferometer has since played a dominant role in this branch of spectroscopy.

Q: - Intensity distribution.

A:- In physics, intensity is a measure of the energy flux, averaged over the period of the wave. The word "intensity" here is not synonymous with "strength", "amplitude", or "level", as it sometimes is in colloquial speech. For example, "the intensity of pressure" is meaningless, since the parameters of those variables do not match.

To find the intensity, take the energy density (that is, the energy per unit volume) and multiply it by the velocity at which the energy is moving. The resulting vector has the units of power divided by area (i.e. W/m^2). It is possible to define the intensity of the water coming from a garden sprinkler, but intensity is used most frequently with waves (i.e. sound or light).

Q:-Define & Describe on Diffraction.

A:-Sound is propagated in the form of waves. Sound produced in an adjoining room reaches us after bending round the edges of the walls.

Similarly, waves on the surface of water also bend round the edges of an obstacle and spread into the region behind it. This bending of waves around the edges of an obstacle is called diffraction. Diffraction is a characteristic property of waves. The waves are diffracted, only when the size of the obstacle is comparable to the wavelength of the wave.

Fresnel showed that the amount of bending produced at an obstacle depends upon the wavelength of the incident wave. Since the sound waves have a greater wavelength, the diffraction effects are pronounced. As the wavelength of light is very small, compared to that of sound wave and even tiny obstacles have large size, compared to the wavelength of light waves, diffraction effects of light are very small.

In practice, diffraction of light can be observed by looking at a source of white light through a fine piece of cloth. A series of coloured images are observed.

Q:-Define Fresnel & Fraunhofer.

A:- Diffraction phenomenon can be classified under two groups (i) Fresnel diffraction and (ii) Fraunhofer diffraction.

In the **Fresnel diffraction**, the source and the screen are at finite distances from the obstacle producing diffraction. In such a case the wave front undergoing diffraction is either spherical or cylindrical.

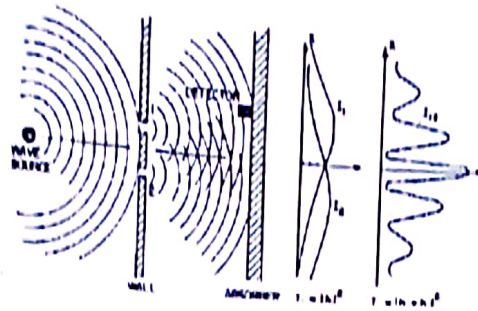
In the **Fraunhofer diffraction**, the source and the screen are at infinite distances from the obstacle producing diffraction. Hence in this case the wavefront undergoing diffraction is plane. The diffracted rays which are parallel to one another are brought to focus with the help of a convex lens. Fraunhofer pattern is easier to observe practically by a spectrometer.

Q:- Diffraction from Two Slits.

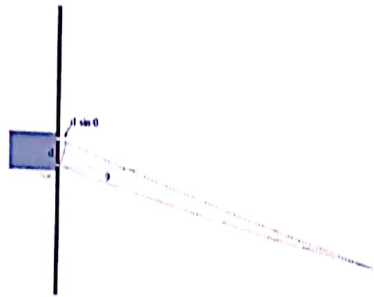
A:- Water waves will exhibit a diffractive interference pattern in a 2 slit experiment as diagrammed below. The diagram shows the crests of the water waves at some time. Downstream from the slits, we will see constructive interference where the waves from the slits are in phase and destructive interference where they are 180 degrees

I_{12}

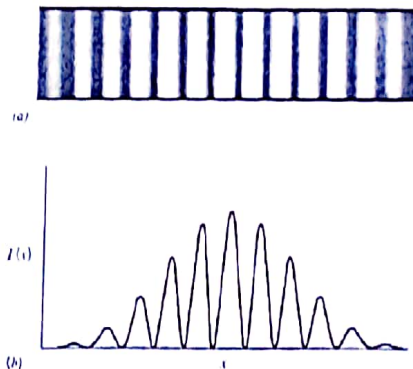
out of phase, for example where a crest from one slit meets a trough from the other. The plot labeled shows the interference pattern at the location of the absorber. Diffraction is a simple wave phenomenon.



The diffraction of light was a well known phenomenon by the end of the 19th century and was well explained in classical Electromagnetic theory since light was held to be a EM wave. The diffraction pattern from two narrow slits is particularly easy to understand in terms of waves. The setup is shown in the diagram below.



EM waves of wavelength λ are emitted from a single light-source, like a laser. They travel to two narrow slits, (for simplicity) equidistant from the source, and a distance d apart. Light travels from the slits to a detection screen. A diffraction pattern can be seen on the detection screen, like the one shown in the picture below.

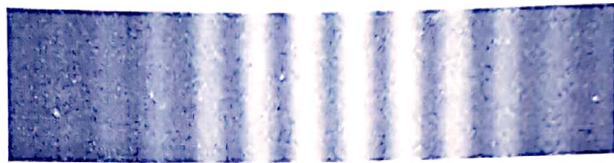


The center of the diffraction pattern occurs at the location on the screen equidistant from each slit where the waves from the two slits are in phase (because they have traveled exactly the same distance) and the fields add, so the waves interfere constructively and there is an intensity maximum. Some distance off this center of the diffraction pattern, there will be destructive interference between waves from the two slits and the intensity will

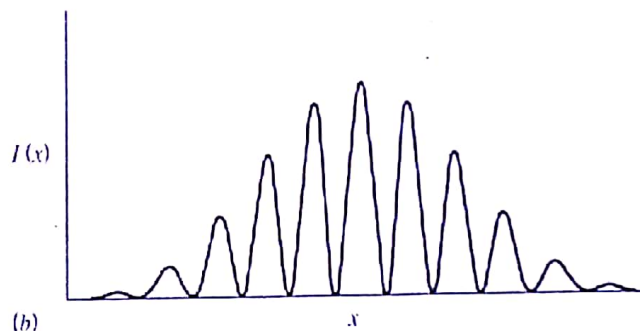
zero. This will occur when the distance traveled by two waves differs by $\lambda/2$, so the waves are 180 degrees out of phase and the fields from the two slits cancel. We can compute this location by looking at the above diagram. We assume that the distance to the screen is much greater than d . For light detected at an angle θ , the extra distance traveled from slit 1 is just $d \sin \theta$. So the angle of the first minimum (or null) can be found from the equation $d \sin \theta = \frac{\lambda}{2}$.

More generally we will get a maximum if the paths from the slits differ by an integer number of wavelengths $d \sin \theta = n\lambda$ and we will get a null when the paths differ by a half integer number of wavelengths.

Although it is very difficult because electrons are charged, 2 slit electron diffraction has also been observed.



(a)



(b)

So, all kinds of particles seem to diffract indicating there is some kind of wave involved. We will now continue with some thought experiments on diffraction to illustrate the physics that Quantum Mechanics needed to match.

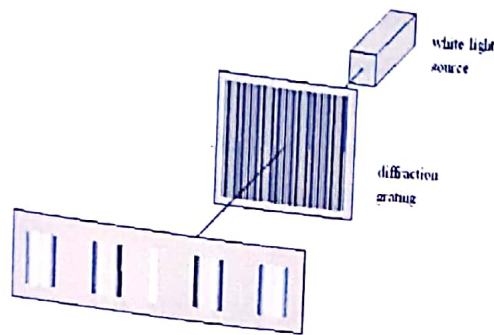
* Example: Derive the location of the nodes in the diffraction pattern from two narrow slits a distance d apart. Now try to compute the intensity distribution.

Q: - Diffraction on multiple slits.

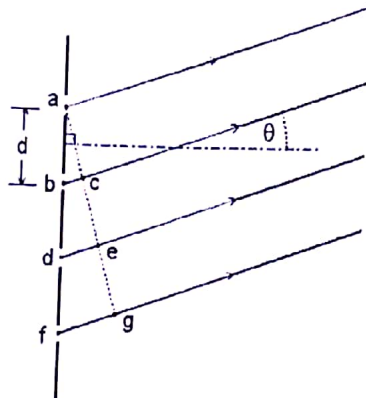
A: - Interference pattern of the light diffracted on multiple slits

- In technology, of immense significance are interference patterns that are formed after diffraction on multiple parallel slits.

- For one, this is the main technique to split the light into spectral components according to the wavelengths used in modern spectrographs.



What is the distribution of the electric field on the screen between diffraction grating that consists of N parallel slits, each of width a and separated by distance d ?



Calculation of the intensity distribution in diffraction grating

- It is rather easy again to calculate (but see textbook for low math discussion) in far-field Fraunhofer regime
- We have learned that a single slit in far field regime creates the distribution of the electric field given by

$$E = E_0 \frac{\sin(\beta/2)}{\beta/2} \cos(kr - \omega t) \quad , \quad \beta = 2\pi \frac{a}{\lambda} \sin \theta$$

- The total electric field from N slits in a grating is just the sum of individual contributions
- Because the sources are coherent, this sum results in interference of individual contributions.
- We must take into account relative phases of the contributions from individual slits to a point on screen. The formula above was written so that we took the phase of the light that passes through the center of our single slit to be zero. Then each subsequent slit has an additional phase shift due to light path length difference. So the contribution of individual slit number i should be written as

$$E_i = E_0 \frac{\sin(\beta/2)}{\beta/2} \cos(kr - \omega t + \Delta\phi_i) \quad \text{and the total is just} \quad E = \sum_{i=1}^N E_i$$

- We shall use the following setup: we consider even number of slits, with a first pair of slits (a and b) at distances $\pm d/2$ from the central line, the second pair at distances $\pm 3d/2$ and so on.
- The phase difference between paths from the centers of slits and the central line are, pair wise $\Delta\phi_1 = \pm \phi/2, \Delta\phi_2 = \pm 3\phi/2, \dots, \Delta\phi_p = \pm (p-1/2)\phi$ & $\phi \equiv 2\pi (d/\lambda) \sin\theta$
Where p enumerates the pairs and ϕ is the phase difference in two-slit interference experiment.
- Now to sum over all slits, we sum the contributions of two slits in each pair, and then sum over all the pairs

$$E = E_0 \frac{\sin(\beta/2)}{\beta/2} \sum_{p=1}^{N/2} [\cos(kr - \omega t + \Delta\phi_p) + \cos(kr - \omega t - \Delta\phi_p)]$$

$$E = 2E_0 \frac{\sin(\beta/2)}{\beta/2} \left[\sum_{p=1}^{N/2} \cos(\Delta\phi_p) \right] \cos(kr - \omega t)$$

- So for intensity

$$I = 4I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \left[\sum_{p=1}^{N/2} \cos(\Delta\phi_p) \right]^2$$

- Or plugging everything in

$$N \text{ slits: } I = 4I_0 \left[\frac{\sin\left(\pi \frac{a}{\lambda} \sin\theta\right)}{\frac{\pi a}{\lambda} \sin\theta} \right]^2 \left[\sum_{p=1}^{N/2} \cos\left((2p-1)\pi \frac{d}{\lambda} \sin\theta\right) \right]^2$$

Q: - What is the Resolution of images.

A: - Image resolution can be measured in various ways. Basically, resolution quantifies how close lines can be to each other and still be visibly resolved. Resolution units can be tied to physical sizes (e.g. lines per mm, lines per inch), to the overall size of a picture (lines per picture height, also known simply as lines, TV lines, or TVL), or to angular subtendant. Line pairs are often used instead of lines; a line pair comprises a dark line and an adjacent light line. A line is either a dark line or a light line. A resolution 10 lines per millimeter means 5 dark lines alternating with 5 light lines, or 5 line pairs per millimeter (5 LP/mm). Photographic lens and film resolution are most often quoted in line pairs per millimeter.

Resolution is the capability of the sensor to observe or measure the smallest object clearly with distinct boundaries. There is a difference between the resolution and a pixel. A pixel is actually a unit of the digital

Resolution depends upon the size of the pixel. The smaller the size of the pixel, the higher the resolution will be and the more clear the object in the image will be. Images having smaller pixel sizes occupy more space on the disk.

The term resolution is often used for a pixel count in digital imaging, even though American, Japanese, and international standards specify that it should not be so used, at least in the digital camera field.

Color images distinguish light of different spectra. Multispectral images resolve even finer differences of spectrum or wavelength than is needed to reproduce color. That is, multispectral images have higher spectral resolution than normal color images.

Radiometric resolution determines how finely a system can represent or distinguish differences of intensity, and is usually expressed as a number of levels or a number of bits, for example 8 bits or 256 levels that is typical of computer image files. The higher the radiometric resolution, the better subtle differences of intensity or reflectivity can be represented, at least in theory. In practice, the effective radiometric resolution is typically limited by the noise level, rather than by the number of bits of representation.

Q: - Describe the Rayleigh criterion.

A: - The Rayleigh criterion is the generally accepted criterion for the minimum resolvable detail - the imaging process is said to be diffraction-limited when the first diffraction minimum of the image of one source point coincides with the maximum of another.

If all parts of an imaging system are considered to be perfect, then the resolution of any imaging process will be limited by diffraction. Considering the single slit expression above, then when the wavelength is equal to the slit width, the angle for the first diffraction minimum is 90° . This means that the wave is spread all the way to the plane of the slit and will not contain resolvable information about the source of the wave. This leads to the simplified statement that the limit of resolution of any imaging process is going to be on the order of the wavelength of the wave used to image it.

Q: - Resolving power of a diffraction grating.

A: - The diffraction grating, a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A transmission grating can be made by cutting parallel lines on a glass plate with a precision ruling machine. The spaces between the lines are transparent to the light and hence act as separate slits. A reflection grating can be made by cutting parallel lines on the surface of a reflective material. The reflection of light from the spaces between the lines is specular, and the reflection from the lines cut into the material is diffuse. Thus, the spaces between the lines act as parallel sources of reflected light, like the slits in a transmission grating. Gratings that have many lines very close to each other can have very small slit spacings. For example, a grating ruled with 5 000 lines/cm has a slit spacing $d = (1/5\,000) \text{ cm} = 2.00 \times 10^{-4} \text{ cm}$.

Resolving power of the diffraction grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise if one wants to distinguish two closely spaced wavelengths. For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda} \quad (38.11)$$

Where $\lambda = (\lambda_1 + \lambda_2) / 2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Thus, a grating that has a high resolving power can distinguish small differences in wavelength. If N lines of the grating are illuminated, it can be shown that the resolving power in the m th-order diffraction is

$$R = Nm \quad (38.12)$$

Thus, resolving power increases with increasing order number and with increasing number of illuminated slits.

Note that $R = 0$ for $m = 0$; this signifies that all wavelengths are indistinguishable for the zeroth-order maximum. However, consider the second-order diffraction pattern ($m = 2$) of a grating that has 5 000 rulings illuminated by the light source. The resolving power of such a grating in second order is $R = 5\,000 \times 2 = 10\,000$. Therefore, for a mean wavelength of, for example, 600 nm, the minimum wavelength separation between two spectral lines that can be just resolved is $\Delta\lambda = \lambda/R = 6.00 \times 10^{-2}$ nm. For the third-order principal maximum, $R = 15\,000$ and $\Delta\lambda = 4.00 \times 10^{-2}$ and so on.

One of the most interesting applications of diffraction is holography, which is used to create three-dimensional images found practically everywhere, from credit cards to postage stamps.

Q: - What are half period zones?

A: - A Fresnel zone plate is a cheapo sort of lens, made of transparent material with a set of concentric black rings. These are sized such that there is a half wavelength path difference (from plate to view screen) between the inner and outer part of a black ring, and also between the inner and outer part of the clear space between the black rings. Light from a point source behind the zone plate will be focused at a point on the view screen. You can get cheap magnifiers which use the same idea.

Fresnel's assumed that a wave front started from a source can be divided in to a

- Large no. of strips or zone called the Fresnel's zone of small area. if the path
- difference between secondary wavelets reaching a point on screen started from
- Two consecutive zone is $N/2$ (or the phase difference is 2π) then the zones are called Fresnel's half period zone..

Q: - What is a zone plate?

A: - A zone plate is a way of focusing light, like a lens, but using Fresnel diffraction instead of refraction. This is cool from a theoretical point of view, because it demonstrates the wave nature of light. And it's cool from a photography point of view, because the images produced have a unique glow, with a impressionistic almost-painted look. If you're attracted to a certain lo-fi "old-camera" aesthetic, using a zone plate is a way to get it authentically, rather than faking it in post-processing.

Zone plates are becoming increasingly useful as a way of focusing X-rays. They are used in X-ray telescopes in astronomy and X-ray lithography in experimental printed circuit fabrication. Basically the zone plate is a way of doing non-lens photography similar to the pinhole but that allows more light to get to the film. This makes the "effective" $f\#$ lower allowing for shorter exposures. Practically this means almost 3 stops or $1/8$ the exposure time. Not bad, this would bring F2048 to F256, a vast improvement. This is one of the least understood areas of non-lens photography. It took me quite some time to find the proper equations and

information. Most pinhole books and literature mention them, but don't give enough information for the lay person to make their own. Well no more, the secret is about to be revealed! Do not be put off by the math, as I give real answers and practical ways of doing later on. MATH: diameter of zone = $2 \times \sqrt{\text{focal length} \times \text{zone index} \times \text{wavelength}}$ Ouch! Let's try a table: (green light, zone plates are wavelength sensitive just like the pinhole)

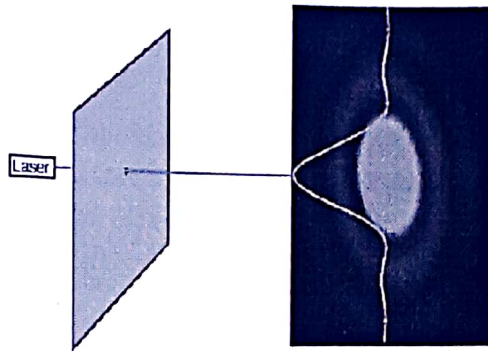
DEFINITION: ZONES

- these are alternating areas of clear (or white) and opaque (or black).

By using a drawing program such as Corel Draw on the PC, we can draw out the Zone Plate at say 100X normal size quite easily.

Q: - Diffraction at a circular aperture.

A: - Circular Aperture Diffraction



When light from a point source passes through a small circular aperture, it does not produce a bright dot as an image, but rather a diffuse circular disc known as Airy's disc surrounded by much fainter concentric circular rings. This example of diffraction is of great importance because the eye and many optical instruments have circular apertures. If this smearing of the image of the point source is larger than that produced by the aberrations of the system, the imaging process is said to be diffraction-limited, and that is the best that can be done with that size aperture. This limitation on the resolution of images is quantified in terms of the Rayleigh criterion so that the limiting resolution of a system can be calculated.

Diffraction by a Straight Edge

Using the slide marked "straight edge/opaque disks," a laser is positioned so that it falls on the edge. The light does go into the shadow region; it bends around the edge and spreads out into the shadow region. The picture below shows what happens when the laser beam falls on a straight edge

Q:-Explain Polarization by Reflection.

A: - Polarization by Reflection: The simplest method of producing plane polarized light is by reflection. Malus, discovered that when a beam of ordinary light is reflected from the surface of transparent medium like glass or water, it gets polarized. The degree of polarization varies with angle of incidence.

Consider a beam of unpolarised light AB, incident at any angle on the reflecting glass surface XY.

Vibrations in AB which are parallel to the plane of the diagram are shown by arrows. The vibrations which are perpendicular to the plane of the diagram and parallel to the reflecting surface, shown by dots (Figure).

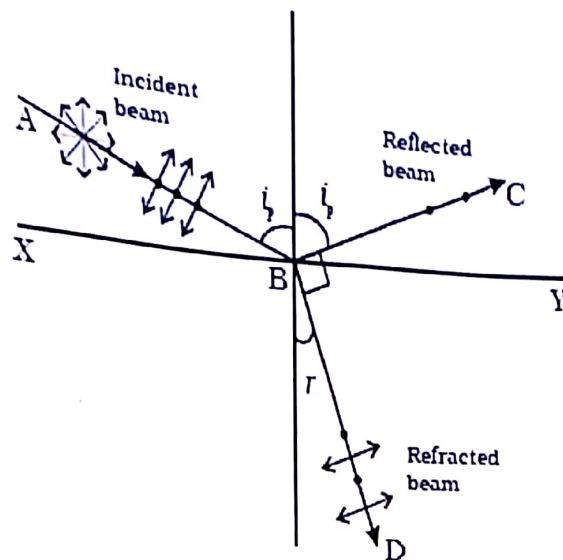


Fig: Polarization by reflection

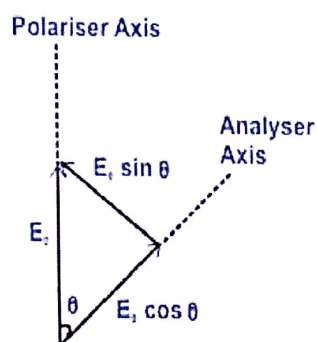
A part of the light is reflected along BC, and the rest is refracted along BD.
On examining the reflected beam with an analyzer, it is found that the ray is partially plane polarized.

When the light is allowed to be incident at a particular angle, (for glass it is 57.5°) the reflected beam is completely plane polarised. The angle of incidence at which the reflected beam is completely plane polarised is called the polarising angle (i_p).

Q: - Malus's law.

A:- According to malus, when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.

i.e $I \propto \cos^2 \theta$



Suppose the angle between the transmission axes of the analyzer and the polarizer is θ . The completely plane polarized light from the polarizer is incident on the analyzer. If E_0 is the amplitude of the electric vector transmitted by the polarizer, then intensity I_0 of the light incident on the analyzer is

$$I \propto E_0^2$$

The electric field vector E_0 can be resolved into two rectangular components i.e $E_0 \cos \theta$ and $E_0 \sin \theta$. The analyzer will transmit only the component (i.e $E_0 \cos \theta$) which is parallel to its transmission axis. However, the component $E_0 \sin \theta$ will be absorbed by the analyser. Therefore, the intensity I of light transmitted by the analyzer is,

$$I \propto (E_0 \times \cos \theta)^2$$

$$I/I_0 = (E_0 \times \cos \theta)^2 / E_0^2 = \cos^2 \theta$$

$$I = I_0 \times \cos^2 \theta$$

Therefore, $I \propto \cos^2 \theta$. This proves law of malus

When $\theta = 0^\circ$ (or 180°), $I = I_0 \cos^2 0^\circ = I_0$ That is the intensity of light transmitted by the analyzer is maximum when the transmission axes of the analyzer and the polarizer are parallel.

When $\theta = 90^\circ$, $I = I_0 \cos^2 90^\circ = 0$ That is the intensity of light transmitted by the analyzer is minimum when the transmission axes of the analyzer and polarizer are perpendicular to each other.

Q: - Define & Describe on Double Refraction.

A: - Bartholinus discovered that when a ray of unpolarised light is incident on a calcite crystal, two refracted rays are produced. This phenomenon is called **double refraction** (Fig: a). Hence, two images of a single object are formed. This phenomenon is exhibited by several other crystals like quartz, mica etc.

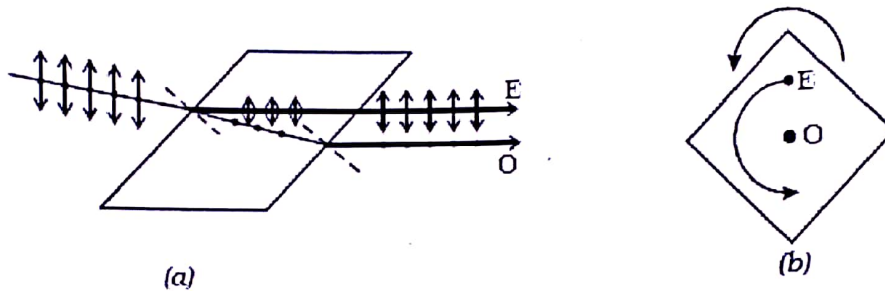
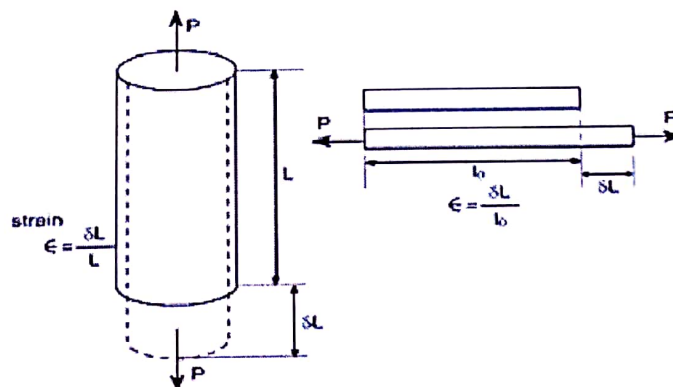


Fig: Double refraction

When an ink dot on a sheet of paper is viewed through a calcite crystal, two images will be seen (Fig: b). On rotating the crystal, one image remains stationary, while the other rotates around the first. The stationary image is known as the ordinary image (O), produced by the refracted rays which obey the laws of refraction. These rays are known as ordinary rays. The other image is extraordinary image (E), produced by the refracted rays which do not obey the laws of refraction. These rays are known as extraordinary rays.



Inside a double refracting crystal the ordinary ray travels with same velocity in all directions and the extraordinary ray travels with different velocities along different directions.

A point source inside a refracting crystal produces spherical wave front corresponding to ordinary ray and elliptical wave front corresponding to extraordinary ray.

Inside the crystal there is a particular direction in which both the rays travel with same velocity. This direction is called optic axis. The refractive index is same for both rays and there is no double refraction along this direction.

Q:- Refraction in Uniaxial crystals

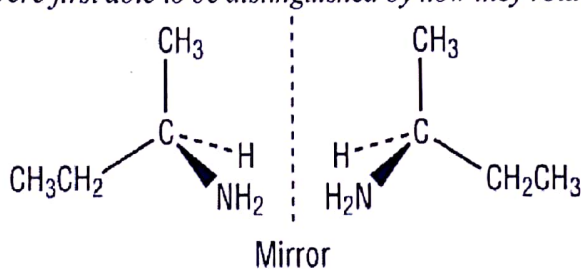
A:- Crystals like calcite, quartz, ice and tourmaline having only one optic axis are called uniaxial crystals.

Double refraction can be observed at an oblique incidence of the light ray onto the crystal surface. Iceland spar and some other crystals contain only one direction along which double refraction does not take place. This direction is called the optic axis of the crystal, and crystals of this type are called uniaxial crystals.

Uniaxial crystals are transmissive optical elements in which the refractive index of one crystal axis is different from the other two crystal axes (i.e. $n_i \neq n_j = n_k$). This unique axis is called the extraordinary axis and is also referred to as the optic axis. Light travels with a higher phase velocity through an axis that has the smallest refractive index and this axis is called the fast axis. Similarly, an axis which has the highest refractive index is called a slow axis since the phase velocity of light is the lowest along this axis. The optic axis can be the fast or the slow axis for the crystal depending upon the material. Negative uniaxial crystals (e.g. calcite CaCO_3 , ruby Al_2O_3) have $n_e < n_o$ so for these crystals, the extraordinary axis (optic axis) is the fast axis whereas for positive uniaxial crystals (e.g. quartz SiO_2 , sellaite (magnesium fluoride) MgF_2 , rutile TiO_2), $n_e > n_o$ and thus the extraordinary axis (optic axis) is the slow axis.

Q:- Optical activity; Rotation of plane of polarization.

A:- Optical isomers are mirror-images to each other and have an asymmetric or chiral carbon atom in their molecules. They can rotate the plane of plane polarized light in the opposite direction. Optical isomers were given their name because they were first able to be distinguished by how they rotated plane-polarized light.



Conditions of Optical Isomerism:

- (i) The compound must be optically active.
- (ii) There must exist a chiral carbon atom in the organic molecule to make it asymmetrical and thus show the optical activity.
- (iii) Optically active compounds rotate the plane of plane polarized light into opposite directions.
- (iv) This results in two different configurations, example; it produces two optical isomers.
- (v) The configurations are mirror images of one another.

Q:- Origin of optical rotation in liquids and in crystals.

A: - Optical rotation is the extent (measured in degrees) to which a solution of a molecule will rotate plane-polarized light, as measured by a Polarimeter. Optical rotation is dependent on concentration, path length, temperature, and the wavelength of light used, as well as the solvent.

Liquid Crystals

In a liquid the molecules have no orderly arrangement. The molecules can move with respect to each other in a random manner, so liquids are able to flow. In a solid the particles are present in a highly ordered pattern, but they cannot move from their positions as they are held rigidly in place. Particles in the solid state cannot flow. Certain organic compounds, exhibit properties characteristic of both liquid and solid in a temperature range just above their melting points. These are called **liquid crystals**.

They are fluids but their molecules are arranged in a highly ordered way. At temperatures above this range the order is lost and they become like any other liquid. Liquid crystal displays (LCD's) are found in calculators, wrist watches, TV screens and a host of other objects in use.

Three types of molecular arrangements, known as **nematic**, **smectic** and **cholesteric** liquid crystals have been identified. Several unsymmetrical molecules that have approximately rod-like shapes behave as liquid crystals. Some examples are shown below along with their chemical formulas: 4, 4'-Dimethoxyazobenzene, 4-butyloxy-4-ethanoylazobenzene, Cholesterylbenzene.

All are organic compounds and end in a polar group, e.g., -CN, -OR (R is an alkyl group), -NO₂, -NH₂ etc. Many have flat parts such as a benzene ring.

The three types of liquid crystals differ in the kind of order of the molecules within the crystals. In the **nematic** liquid crystals the molecules are arranged parallel to each other but can slide or roll over each other, but their centres are oriented randomly.

Smectic liquid crystals show greater order. The molecules are parallel to each other and are arranged in layers that are one molecule thick. These are able to slide readily over each other. Liquid crystals of this type are turbid and viscous.

In **cholesteric liquid crystals** the molecules are arranged in thin layers but they are aligned parallel to the layers in the nematic fashion, with slightly different orientations from one layer to the next.

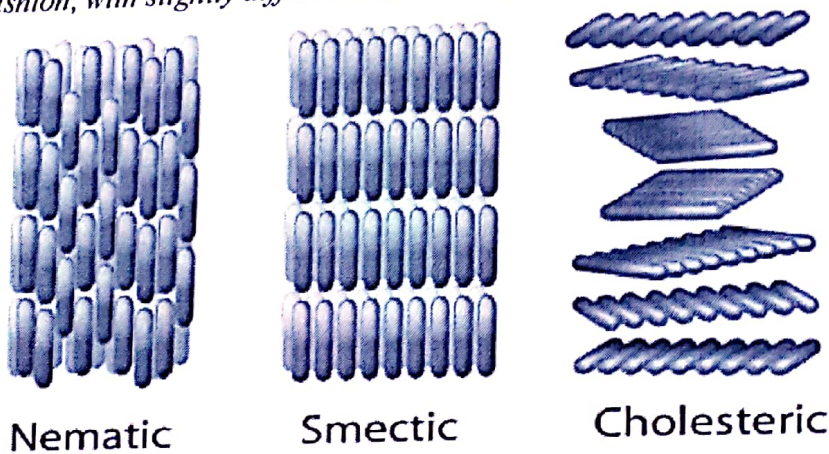


Figure: Arrangements of molecule in liquid crystals (a) Packing in a nematic liquid crystal (b) Packing in a Smectic liquid crystal and (c) Packing in a Cholesteric liquid crystal.

The use of liquid crystals is based on their special electrical and optical properties. In liquid crystal displays (LCD's) a thin layer of nematic liquid crystal is placed between two electrodes (at least one of which is transparent) that are arranged on glass in special pattern. In the absence of any electric signal across the electrodes the molecules are uniformly oriented. In this configuration light passes readily through the cell. When a particular electrode segment is energized, the orientations of the molecules in the liquid crystal are changed and the substance becomes opaque. By activating appropriate segments in this way various numbers and letters can be formed and displayed. One advantage of the liquid crystal devices is that they use very little power.

In cholesteric liquid crystals the presence of smectic-like layers give rise to colors in the light scattered from them. The spacing of the layers depends on the temperature: consequently different colors are observed at different temperatures. This effect is used to make liquid crystal thermometer. In medical science small difference of temperature on different parts of the skin of a patient due to abnormalities can be detected from the color change of liquid crystals. The technique is called thermographs.