# Wave

— Wave motion — Wave motion is a kind of disturbance which travels through a medium due to repeated vibration of the Particles of the medium about their mean Positions, the disturbence being handed over from one Particle to the Next.

In a wave, both information and energy Propagate in the form of a signals from one Point to another but there is no motion of matter as a whole through a medium.

Characteristics —

1. In a wave motion the disturbance travels through the medium due to repeated Periodic oscillations of the Particles of the medium about their mean Position.
2. The energy is transferred from Place to another without any actual transfer of the Particles of the medium.
3. Each Particle receives disturbance a little later than its Preceding Particle. There is a regular Phase difference between one Particle and the next.
4. The velocity with which a wave travels is different from the velocity of the Particles with which they Vibrate about their mean Position.
5. The wave velocity remains Constant in a given medium while the Particle velocity changes continuously during its Vibration about the mean Position. It is maximum at the mean Position and zero at the extreme Position.
6. For the Propagation of a mechanical wave, the medium must Possess the Properties of inertia, elasticity and minimum friction amongst its Particles.

— Types.

a. Mechanical waves — The waves which require a material medium for Propagation are Called mechanical waves. Such waves are also Called elastic waves because

Their Propagation depends on the elastic Properties of the medium

b. Electromagnetic waves :- The waves which travel in the form of oscillating electric and magnetic field are called electromagnetic waves. Such waves donot require any material medium for their Propagation and are also called non-mechanical waves.

c. Matter waves — The waves associated with microscopic particles, such as electrons, Protons, neutrons, atoms, molecules etc. when they are in motion are called matter waves or de-Broglie waves.

Types of wave motion-

1 Transverse waves — these are the waves in which the individual particles of the medium oscillate perpendicular to the direction of wave Propagation. The Point of maximum displacement in the upward direction are called crests. The Point of maximum displacement in the downward direction are called trough. one crests and one trough together form one wave.

2. Longitudinal wave — These are the waves in which the individual particles of the medium oscillate along the direction of wave Propagation.

— Essential Properties of a medium for the Propagation of mechanical waves-

a Elasticity — The medium must possess elasticity so that the particles can return to their mean positions after being disturbed.

b. Inertia — The medium must possess inertia or mass so that its particles can store kinetic energy.

c. Minimum friction. — the frictional force amongest the particles of the medium should be negligibly small so that they continue oscillating for a sufficiently long time and the wave travels a sufficiently long distance through the medium.

- Amplitude – It is the maximum displacement suffered by the particles of the medium about their mean position. It is denoted by A.
- Time Period – The time period of a wave is the time in which a particle of medium completes one vibration to and fro about its mean position. It is denoted by T.
- Frequency – The frequency of a wave is the number of waves produced per unit time in the given medium. It is denoted by $\nu$ and its S.I. unit is $s^{-1}$ or Hertz (Hz)
- Angular frequency – The rate of change of phase with time is called angular frequency of the wave. It is equal to $\frac{2\pi}{T}$. It is denoted by $\omega$ and its S.I. unit is rad/sec.
- Wavelength – It is the distance covered by a wave during the time in which a particle of the medium completes one vibration to and fro about its mean position. or it is the distance between two nearest particle of the medium which are vibrating in the same phase.
- Wave number – The number of waves present in a unit distance of the medium is called wave number. It is equal to the reciprocal of wave length $\lambda$. thus

wave number $\bar{\nu} = \frac{1}{\lambda}$.

- Angular wave number or Propagation Constant. –

The quantity $\frac{2\pi}{\lambda}$ is called angular wave number or Propagation constant of a wave. It represent the phase change per unit path difference. It is denoted by K.

$$K = \frac{2\pi}{\lambda}$$

S.I. unit of K is rad/meter.

- **Wave Velocity or Phase velocity** — The distance Covered by a wave Per unit time in its direction of Propagation is called its wave velocity or Phase velocity. It is denoted by $v$.

- **Relation between wave velocity, frequency, and wavelength:**
when a Particle of the medium Completes one oscillation about its mean Position in Periodic time $T$, the wave travels a distance equal to its wave length $\lambda$ so.

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}}.$$

$$v = \frac{\lambda}{T} = \lambda \nu$$

$$v = \lambda \times \nu$$

Wave velocity = wave length × frequency.

- **Speed of transverse wave on a Stretched String** —
The wave velocity through a medium depends on its inertial and elastic Properties. So the speed of transverse wave through a Stretched String is determined by two factors.

(i) Tension $T$ in the String is a measure of elasticity in the String. Without tension no disturbance can Propagate in the String.

$$T = \text{Force} = MLT^{-2}.$$

(ii) Mass Per unit length or linear mass density $m$ of the String so that the String Can Store kinetic energy.

$$m = \frac{\text{mass}}{\text{Length}} = ML^{-1}$$

Now.

$$\text{Dimension of ratio} = \frac{T}{m} = \frac{MLT^{-2}}{ML^{-1}} = L^2T^{-2}.$$

As the speed $v$ has the dimension ($LT^{-1}$). So we Can express $v$ in term of $T$ and $m$ as.

$$v = C\sqrt{\frac{T}{m}}.$$

The speed of a transverse wave along a streatched string depends only on the tension T and linear mass density m of the string. It does not depend on the frequency of the wave. The frequency of a wave depends on the source generating the wave.

— Speed of transverse wave in a solid — The speed of transverse wave through solid is determined by two factors.

(i) Elasticity of shape or modulus of rigidity $\eta$ of the solid.

(ii) mass per unit volume or density $\rho$ determines its inertia.

Now. $\frac{\eta}{\rho} = \frac{ML^{-1}T^{-2}}{ML^{-3}} = L^2T^{-2}$.

Dimension of $v = (LT^{-1})$

so $v = c\sqrt{\frac{\eta}{\rho}}$

— Speed of a longitudinal wave in a liquid or gas.

$$v = c\sqrt{\frac{k}{\rho}}$$ where $k$ = bulk modulus. $\rho$ = density.

— Speed of longitudinal wave in a solid.

$$v = c\sqrt{\frac{k + \frac{4}{3}\eta}{\rho}}$$

$k$ = bulk modulus.
$\eta$ = modulus of rigidity.
$\rho$ = density.

— Speed of a longitudinal wave in a solid rod.

$$v = \sqrt{\frac{Y}{\rho}}$$

$Y$ – Young modulus.
$\rho$ = density.

— Newton's formula for the speed of sound in a gas — Newton assumed that sound waves travels through a gas under isothermal condition, then the small amount of heat produced in a compression is rapidly conducted to the surrounding rarefactions where slight cooling is produced. thus the temperature of gas remains constant. If $K_{iso}$ is the isothermal volume elasticity (Bulk modulus of gas at constant temperature) then the speed of sound in the gas will be.

$$v = \sqrt{\frac{K_{iso}}{\rho}}$$ $\rho$ - density of gas.

For an isothermal change $PV$ = constant.

Differentiating both sides.

$$PdV + VdP = 0 \quad \text{or} \quad PdV = -VdP$$

$$P = -\frac{VdP}{dV} \quad \text{or} \quad P = -\frac{dP}{dV/V}$$

$$= \frac{\text{volume stress}}{\text{volume strain}} = K_{iso}.$$

$$v = \sqrt{\frac{P}{\rho}}$$

- Laplace's Correction – Laplace pointed out that sound travels through a gas under adiabatic condition not under isothermal condition. Because

a. As sound travel through a gas, temperature rises in the regions of compression and falls in the region of rarefaction.

b. A gas is a poor conductor of heat.

c. The compression and rarefactions are formed so rapidly that the heat generated in the regions of compressions doesnot get time to pass into the region of rarefactions. so as to equalise the temperature.

then.

$$v = \sqrt{\frac{K_{adi}}{\rho}}$$

For an adiabatic change $PV^{\gamma}$ = constant.

Differentiating both side.

$$P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0.$$

$$\gamma P = -\frac{dP}{dV/V} = K_{adia}.$$

where $\gamma = \frac{C_P}{C_V}$

then $v = \sqrt{\frac{\gamma P}{\rho}}$

Factors affecting speed of sound in a gas.

1 Effect of Pressure – The speed of sound

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature $PV = $ constant

$\frac{Pm}{\rho} = $ constant. or $\frac{P}{\rho} = $ constant.

When Pressure changes, density also changes in the same ratio so that the factor $\frac{P}{\rho}$ remains unchanged. Hence Pressure has no effect on the speed of sound in a gas.

(II) Effect of density – Suppose two gases have the same Pressure $P$ and same value of $\gamma$. If $\rho_1$ and $\rho_2$ are the densities of the two gases then the speed of sound in them.

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \qquad v_2 = \sqrt{\frac{\gamma P}{\rho_2}}.$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Hence at constant Pressure, the speed of sound in a gas is inversely Proportional to the square root of its density.

(III) Effect of humidity – Because of $v \propto \frac{1}{\sqrt{\rho}}$ so speed of sound is faster in moist air than in dry air.

(iv) Effect of temperature — For one mole of a gas $PV = RT$

If $M$ is the molecular mass of the gas and $\rho$ its density then. $\rho = \frac{M}{V}$ or $V = \frac{M}{\rho}$

$$\therefore \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M}$$

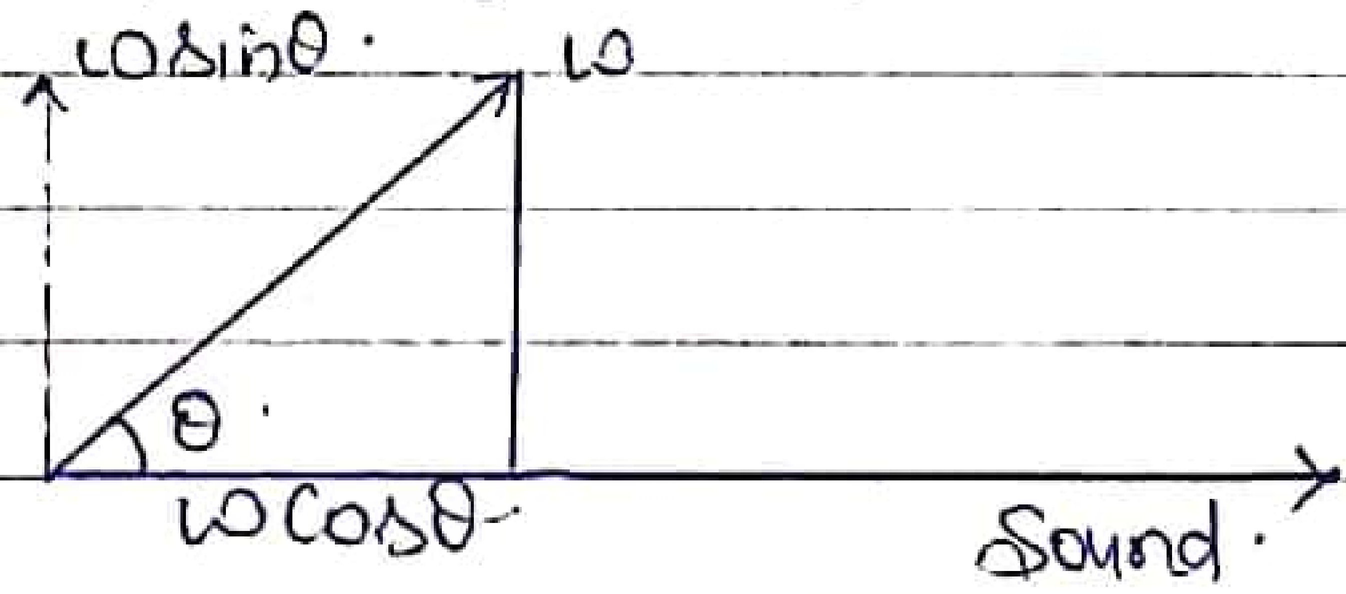
$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad \text{or} \quad v \propto \sqrt{T}.$$

Hence speed of sound in a gas is directly proportional to the square root of its absolute temperature.

The velocity of sound in air increases by 61 cm/sec. for every 1°C rise in temperature. This is known as the temperature coefficient for sound in air.

(V) Effect of wind — Suppose the wind travels with velocity $w$ at angle $\theta$ with the direction of propagation of sound.

The component of wind velocity in the direction of sound is $w\cos\theta$.




then Resultant velocity of Sound $= v + w\cos\theta$

when the wind make $\theta = 0$ then $\cos 0 = 1$

then Resultant velocity $= v + w$.

If $\theta = 180°$ then. $\cos 180 = -1$

Resultant velocity $= v - u$.

(VI) Effect of frequency — The speed of sound in air is independent of its frequency

(VII) Effect of amplitude — The speed of sound is independent of the amplitude of the sound wave. But if the amplitude is very large, the compression and rarefaction may cause.

large temperature variations which may affect the speed of sound.

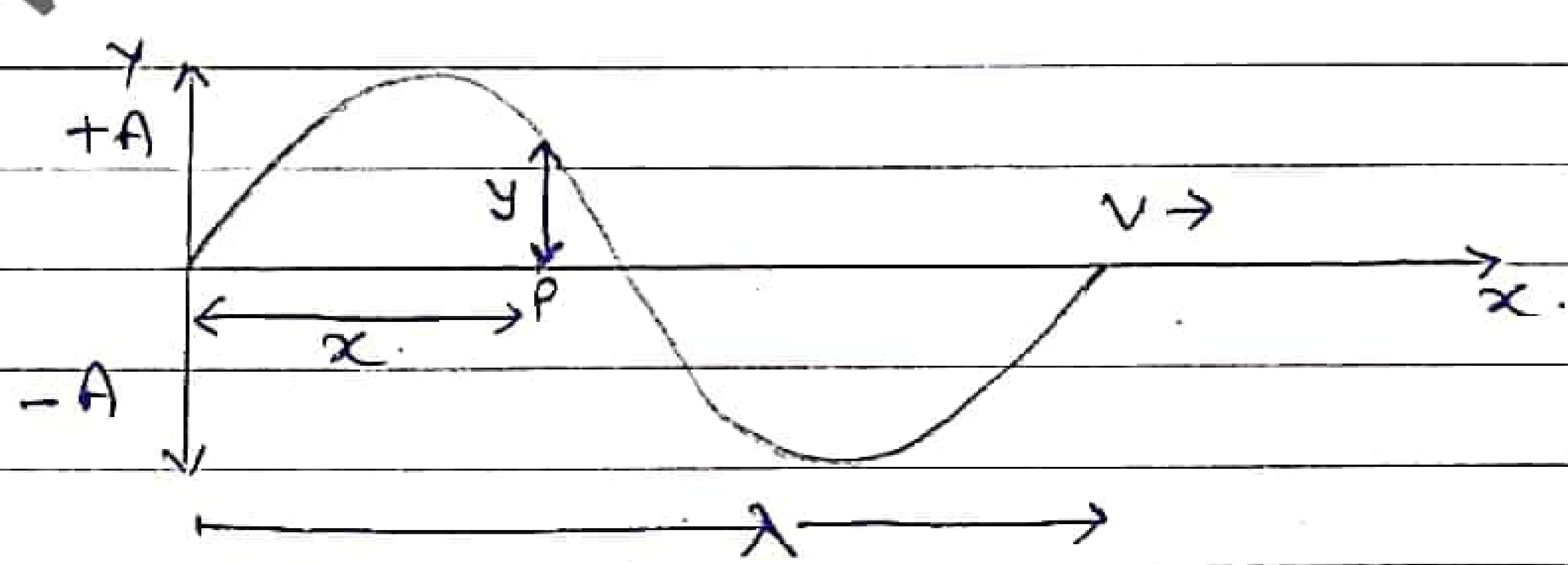
- Progressive wave – A wave that travel from one point of the medium to another is called a progressive wave. A progressive wave may be transverse or longitudinal.

- Plane progressive harmonic wave – If during the propagation of a wave through a medium, the particles of the medium vibrate S.H. about their mean position. then the wave is said to be plane progressive harmonic wave.

- Displacement relation for a progressive harmonic wave – Suppose a S.H. wave starts from the origion (O) and travel along the positive direction of x-axis with speed v. Let the time be measured from the instant when the particle at the origion O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at the origion O ($x=0$) at any instant t is given by

$$y(0,t) = A\sin\omega t.$$

Where T is the periodic time and A is amplitude.




Consider a particle P on the x-axis at a distance x from O. The disturbance starting from the origin O will reach P in $\frac{x}{v}$ second. This means the particle P will start vibrating $\frac{x}{v}$ second later than the particle at O. therefore.

Displacement of the Particle P at any instant t.

= Displacement of the Particle at O at a time $x/v$ second earlier.

= Displacement of the Particle at O at time $\left(t-\frac{x}{v}\right)$.

then $y(x,t) = A\sin\omega\left(t-\frac{x}{v}\right) = A\sin\left(\omega t - \frac{\omega x}{v}\right)$

But $\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$ $\quad \because v = \nu\lambda$.

The Quantity $k = \frac{2\pi}{\lambda}$ is called angular wave number or Propagation Constant.

so $y(x,t) = A\sin(\omega t - kx)$

This equation represent a harmonic wave travelling along the Positive direction of x-Axis. It can also be written in the following forms.

$$y(x,t) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}.x\right)$$

$$= A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$= A\sin\frac{2\pi}{T}\left(t - \frac{x}{\lambda}T\right)$$

$$= A\sin\frac{2\pi}{T}\left(t - \frac{x}{v}\right) \qquad \because \frac{\lambda}{T} = v$$

A Harmonic wave travelling along negative direction of x-Axis is

$$y(x,t) = A\sin(\omega t + kx + \phi_0)$$

Along Positive direction of x-Axis.

$$y(x,t) = A\sin(\omega t - kx + \phi_0)$$

— Phase of a wave – The Phase of a harmonic wave is a Quantity that gives Complete information of the wave at any time and at any Position. It is equal to the argument of the sine or cosine function representing the wave.

Suppose

$$Y(x,t) = A\sin(\omega t - kx + \phi_0)$$

then the Phase of the wave at Position $x$ and time $t$ is

$$\phi = \omega t - kx + \phi_0 \quad \text{— (1)}$$

- Taking $x$ as constant, If we differentiate the eq. (1) w.r.t. of time $t$

then $\frac{d\phi}{dt} = \omega$

then $\Delta\phi = \omega\Delta t = \frac{2\pi}{T}.\Delta t$.

Hence we can define the time Period of a wave as the time in which the Phase of a Particle of the medium changes by $2\pi$.

- Taking $t$ as constant, If we differentiate eq. (1) w.r.t. to Position $x$ then

$\frac{\Delta\phi}{\Delta x} = -k$. or $\Delta\phi = -k.\Delta x$

$= \frac{-2\pi}{\lambda}\Delta x$.

we can define the wave length of a wave as the distance between two Points which have a Phase difference of $2\pi$. at any given instant. The negative sign indicates the farther the Particle is located from the origin in the Positive $x$-direction, the more it legs behind in Phase.

- Particle velocity — It is the velocity with which the Particles of the medium vibrate about their mean Positions.

$$Y(x,t) = A\sin(\omega t - kx) \quad \text{— (1)}$$

$$V = \frac{dY}{dt} = \omega A\cos(\omega t - kx) \quad \text{— (2)}$$

$$= \omega A\sin[(\omega t - kx) + \pi/2]$$

(i) While the wave velocity ($v = \nu\lambda$) remains constant, the Particle velocity change S.H. with time.

(ii) The Particle velocity is ahead of displacement in Phase by $\pi/2$ radian.

(III) The maximum Particle velocity or the velocity amplitude is

$$v_0 = \omega A = \frac{2\pi}{T} \times A$$

(IV) If we differentiate eq. ① w.r.t to $x$.

$$\frac{dy}{dx} = -KA\cos(\omega t - kx) \quad \text{— ②}$$

By eq. ② and ③.

$$\frac{v}{dy/dx} = \frac{A\omega\cos(\omega t - kx)}{-KA\cos(\omega t - kx)}$$

$$= -\frac{\omega}{K} = \frac{-2\pi\nu}{2\pi/\lambda} = -\nu\lambda = -v.$$

or $v = -v\frac{dy}{dx}$. where $v$ = wave velocity.

then Particle velocity at a Point = −wave velocity × slop of displacement curve at that Point

— Particle acceleration —

$$v = \omega A\cos(\omega t - kx).$$

$$a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t - kx)$$

$$= -\omega^2 y.$$

or $a = \omega^2 A\sin[(\omega t - kx) + \pi]$

(V) The maximum velue of Particle acceleration or the accelerat-ion amplitude is.

$$a_0 = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A.$$

(VI) The Particle acceleration is ahead of the Particle displace-ment in Phase by $\pi$ radian.

— wave velocity or Phase Velocity — The distance covered by a wave in the direction of its Propagation Per unit time is Called the wave velocity.

— Reflection of a wave from a rigid boundary —

Consider a wave pulse travelling along a string attached to a rigid support. As the pulse reaches the support, it exerts an upward or downward force on the support. By Newton's third law, the support exerts an equal downward or upward force on the string. this produce a reflected pulse in the downward or upward direction, which travel in the reverse direction. Hence when a travelling wave is reflected from a rigid boundary, it is reflected back with a phase reversal or phase difference of $\pi$ radians.

Suppose an incident wave is represented by

$$y(x,t) = A\sin(\omega t - kx).$$

then reflected wave. $y(x,t) = A\sin(\pi + (\omega t - kx))$

so. $y(x,t) = -A\sin(\omega t + kx)$

— Reflection of a wave from an open boundary —

when a travelling wave is reflected from a free or open boundary, it suffers no phase change.

then Reflected wave.

$$y(x,t) = A\sin(\omega t + kx)$$

— Independent behaviour of waves — when a number of waves travel through a region at the same time, each wave travel independently of the others.

— Principle of superpositions of waves — when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebric sum of the displacement due to the individual waves.

The principle of superposition holds not only for the mechanical waves but also for electromagnetic waves.

- Stationary waves – When two identical waves of same amplitude and frequency travelling in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called stationary wave or standing wave.

The resultant wave keeps on repeating itself in the same fixed position. Some particles of the medium remains permanently at rest, they have zero displacement so their position are called nodes, some other particles suffers maximum displacement, their position are called antinodes.

a. Transverse stationary wave – when two identical transverse waves travelling in opposite directions overlap, a transverse stationary wave formed.

b. Longitudinal stationary wave – when two identical longitudinal waves travelling in opposite directions overlap, a longitudinal stationary wave is formed.

- Analytical treatment of stationary waves.

Consider two waves of equal amplitude and frequency travelling along a long string in opposite directions.

The wave travelling along positive x-direction.

$$y_1 = A\sin(\omega t - kx).$$

The wave travelling along negative x-direction.

$$y_2 = A\sin(\omega t + kx).$$

then: According to the principle of superposition, the resultant wave.

$$y = y_1 + y_2 = A\left(\sin(\omega t + kx) + \sin(\omega t - kx)\right)$$

$$= A\,(2\sin\omega t\cos kx)$$

$$= (2A\cos kx)\sin\omega t.$$

this equation represent a stationary wave.

change with Position $x$.

1. When the amplitude will be zero

then $2A\cos kx = 0$ or $\cos kx = 0$.

then $kx = (n+\frac{1}{2})\pi$. where $n = 0, 1, 2, \ldots$

$$\frac{2\pi}{\lambda}x = (n+\frac{1}{2})\pi \qquad \because k = \frac{2\pi}{\lambda}.$$

$$\text{or } x = (2n+1)\frac{\lambda}{4} \qquad \text{or } x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}.$$

These Position of zero amplitude are called nodes. the seperation between two consecutive nodes is $\lambda/2$

2. When the amplitude will have a maximum value of $2A$.

then:

$$2A\cos kx = \pm 2A \quad \text{or } \cos kx = \pm 1$$

$$kx = n\pi$$

$$\frac{2\pi}{\lambda}x = n\pi. \quad \text{or } x = \frac{n\lambda}{2}.$$

$$x = 0, \lambda/2, \lambda, \frac{3\lambda}{2} \ldots$$

These Position of maximum amplitude are called antinodes and two antinodes seperates by $\lambda/2$.

— Change with time.

a. At the instant $t = 0, T/2, 3T/2 \ldots$

$$\sin\omega t = \sin\frac{2\pi}{T}\cdot t = 0.$$

At these instants the displacement $y$ become zero at all the Points. All the Particles of the medium Pass through their mean Positions

b. At the instant $t = T/4, 3T/4, 5T/4 \ldots$

$$\sin\omega t = \sin\frac{2\pi}{T}\cdot t = \pm 1$$

At these instants the displacement $y$ is maximum at all.

all the Points and becomes alternately Positive and negative.

— The Superposition of two waves may lead to following three different effects.

1. When two waves of the same frequency moving with the same speed in the same direction in a medium superpose on each other, they give rise to effect called interference of waves.
2. When two waves of same frequency moving with the same speed in the opposite directions in a medium superpose on each other they produce stationary waves.
3. When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each other, they produce beats.

- Superposition of two identical pulse travelling in opposite direction make constructive interference.
- Superposition of two equal and opposite pulses travelling in opposite direction make destructive interference.

— let incident wave $y_1 = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

reflected wave

$$y_2 = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

then stationary wave formed by the superposition is

$$y = y_1 + y_2$$

$$= \pm 2A\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T}$$

Case I

For (+) sign, antinodes are formed at the Position

$$x = 0, \lambda/2, \lambda, 3\lambda/2 \ldots\ldots$$

and nodes are formed $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots\ldots$

Case (II)

For (−) sign, Antinodes are formed $x = \lambda/4, 3\lambda/4, 5\lambda/4 \dots$

nodes are formed $x = 0, \lambda/2, 3\lambda/2, \dots$

## Difference

| | Progressive wave | Stationary wave. |
|---|---|---|
| 1 | The disturbance travels forward with a definite velocity | The disturbance remains confine to the region where it is produced. |
| 2. | Each particle of the medium executes S.H.M. with same amplitude. | Except nodes, all particle of the medium execute S.H.M. with varying amplitude. |
| 3. | There is a continuous change of phase from one particle to the next. | All the particles between two successive nodes vibrate in the same phase, but the phase reverse for particles between next pair of nodes. |
| 4. | No particle of the medium is permanently at rest | The particle of the medium at nodes are permanently at rest. |
| 5. | There is no instant when all the particles are at the mean position together. | Twice, during each cycle, all particles pass through their mean position. |
| 6. | There is a flow of energy across every plane along the direction of propagation of the wave. | Energy of one region remains confined in that region. |
| 7. | The energy averaged over a wavelength is half kinetic and half potential. | Twice during each cycle, the energy become alternately wholly potential or kinetic. |
| 8. | All the particles have same maximum velocity which they attain while passing their mean position one after other. | All the particles attain their individual maximum velocity at the same time as they pass through their mean position. |

– Laws of transverse vibrations of a string – the fundamental frequency produced in a stretched string of length L under tension T and having mass per unit length m is given by $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$

This equation gives the following laws of vibrations of strings.

1. Law of length – The fundamental frequency of a vibrating string is inversely proportional to its length provided its tension and mass per unit length remain the same.

$$\nu \propto \frac{1}{L} \quad (T \text{ and } m \text{ are constant})$$

2. Law of tension – The fundamental frequency of a vibrating string is proportional to the square root of its tension

$$\nu \propto \sqrt{T}. \quad (L \text{ and } m \text{ are constant})$$

3. Law of mass – The fundamental frequency of a vibrating string is inversely proportional to the square root of its mass per unit length.

$$\nu \propto \frac{1}{\sqrt{m}}. \quad (L \text{ and } T \text{ are constant})$$

If $\rho$ is the density of the string and $D$ its diameter then its mass per unit length will be

$m$ = volume of unit length × density,

$$= \pi \cdot \frac{D^2}{4} \times \rho$$

$$\therefore \nu = \frac{1}{2L}\sqrt{\frac{T \times 4}{\pi D^2 \rho}} = \frac{1}{LD}\sqrt{\frac{T}{\pi \rho}}$$

4. Law of diameter – the fundamental frequency of a vibrating string is inversely proportional to its diameter

$$\nu \propto \frac{1}{D} \quad (L, T, \text{ and } \rho \text{ Constant})$$

5 Law of density – The fundamental frequency of a vibrating string is inversely proportional to the square root of its density.

$$\nu \propto \frac{1}{\sqrt{\rho}} \qquad (T, L \text{ and } D \text{ constant})$$

If a string vibrate in $P$ segments then-

$$\nu = \frac{P}{2L}\sqrt{\frac{T}{m}}$$

– Organ Pipe – It is the simplest musical instrument in which sound is produced by setting an air column into vibration.

Q. Prove analytically treatment of stationary wave in a string fixed at both ends.

An. Consider a uniform string of length $L$ stretched by a tension $T$ along the $x$-axis, with its ends rigidly fixed at the end $x=0$ and $x=L$.

Suppose a transverse wave produced in the string travels along the string along positive $x$-direction and get reflected at the fixed ends $x=L$ the two waves can be represented as

$$y_1 = A\sin(\omega t - kx).$$

$$y_2 = -A\sin(\omega t + kx).$$

then

$$y = y_1 + y_2 = A\left(\sin(\omega t - kx) - \sin(\omega t + kx)\right)$$

$$= -2A\cos\omega t \sin kx. \qquad -(1)$$

we have the boundary conditions.

$y=0$ at $x=0$ for all $t$.

$y=0$ at $x=L$ for all $t$.

The first condition is satisfied by eq. (1). The second condition will be satisfied if.

$$y = -2A\sin kL\cos\omega t = 0.$$

this will be true, for all value of $t$ only if.

$\sin KL = 0$ or $KL = n\pi$. where $n = 1, 2, \ldots$

$\frac{2\pi L}{\lambda} = n\pi$. or $\lambda = \frac{2L}{n}$

The speed of the transverse wave on a string of liner mass density $m$ is

$$v = \sqrt{\frac{T}{m}}.$$

then the frequency of the vibrating string is.

$$\nu = \frac{v}{\lambda} = \frac{n}{2L}\sqrt{\frac{T}{m}}.$$

For $n = 1$

$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$ is called fundamental frequency.

– Analytical treatment of stationary wave in an open organ pipe – Consider a cylindrical pipe of length $L$ lying along $x$-axis, with its open end at $x = 0$ and $x = L$. The sound wave travelling along the pipe may be

$$y_1 = A\sin(\omega t - kx).$$

The wave reflected from open end may be

$$y_2 = A\sin(\omega t + kx)$$

By the principle of superposition.

$$y = y_1 + y_2 = A(\sin(\omega t - kx) + \sin(\omega t + kx))$$
$$= 2A\sin\omega t\cos kx.$$
$$= (2A\cos kx)\sin\omega t.$$

For all values of $t$, the resultant displacement is maximum (+ or −) or antinodes are formed at the open ends at $x = 0$ and $x = L$. This condition is satisfied if.

$\cos KL = \pm 1$ or $KL = n\pi$

$\frac{2\pi}{\lambda} \times L = n\pi$. or $\lambda = \frac{2L}{n}$.

The frequency of vibration. $\nu = \frac{n}{2L}\sqrt{\frac{YP}{\rho}}$

— Analytical treatment of stationary wave in a closed organ pipe – Consider a cylindrical pipe of length L lying along the x-axis with its closed end at $x=0$ and open end at $x=L$. The sound wave sent along the pipe may be

$$y_1 = A\sin(\omega t + kx).$$

The wave reflected from the closed end.

$$y_2 = -A\sin(\omega t - kx).$$

By the principle of superposition. the resultant stationary wave.

$$y = y_1 + y_2 = A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$
$$= 2A\cos\omega t\sin kx$$
$$= (2A\sin kx)\cos\omega t$$

At $x=0$, $y=0$ so a node is formed at the closed end. The resultant displacement at $x=L$ will be maximum if. $\sin kL = \pm 1$ or $kL = (2n-1)\frac{\pi}{2}$.

$$\frac{2\pi}{\lambda}L = (2n-1)\frac{\pi}{2} \quad \text{or} \quad \lambda = \frac{4L}{2n-1}$$

The corresponding frequency of vibration. $\nu = \frac{v}{\lambda} = \frac{2n-1}{4L}\sqrt{\frac{YP}{\rho}}$

— Beats – when two sound waves of slightly different frequency travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls alternately with time. These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequency are called beats. one rise and one fall constitute one beat. The number of beats produced per second is called beat frequency. For beats to be audible, the diffence in the frequency of the two sound waves should not exceed 10.

**Analytical treatment of waves** – Consider two harmonic waves of frequencies $\nu_1$ and $\nu_2$ ($\nu_1 > \nu_2$) and each of amplitude A travelling in a medium in the same direction. The displacement due to the two waves

$$y_1 = A\sin\omega_1 t = A\sin 2\pi\nu_1 t.$$
$$y_2 = A\sin\omega_2 t = A\sin 2\pi\nu_2 t.$$

By the Principle of superposition, the resultant displacement.

$$y = y_1 + y_2 = A[\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t]$$
$$= 2A\cos 2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t . \sin 2\pi\left(\frac{\nu_1+\nu_2}{2}\right)t.$$

Let $\frac{\nu_1-\nu_2}{2} = \nu_{mod}$ and $\frac{\nu_1+\nu_2}{2} = \nu_{av}.$

then $y = 2A\cos(2\pi\nu_{mod} t) . \sin(2\pi\nu_{av} . t)$
$= R . \sin 2\pi\nu_{av} t$

where $R = 2A\cos(2\pi\nu_{mod} t)$ is the amplitude of the resultant wave. The amplitude R of the resultant wave will be maximum when.

$$\cos 2\pi\nu_{mod} t = \pm 1$$
$$2\pi\nu_{mod} t = n\pi \quad \text{or} \quad \pi(\nu_1-\nu_2)t = n\pi.$$
$$t = \frac{n}{\nu_1-\nu_2} = 0, \frac{1}{\nu_1-\nu_2}, \frac{2}{\nu_1-\nu_2}, \ldots\ldots$$

Time interval between two successive maxima. $= \frac{1}{\nu_1-\nu_2}.$

Similarly, the amplitude R will be minimum. when.

$$\cos 2\pi\nu_{mod} . t = 0 \quad \text{or} \quad 2\pi\nu_{mod} t = (2n+1)\pi/2.$$
$$\pi(\nu_1-\nu_2) = (2n+1)\pi/2.$$
$$t = \frac{2n+1}{2(\nu_1-\nu_2)} = \frac{1}{2(\nu_1-\nu_2)}, \frac{3}{2(\nu_1-\nu_2)}, \frac{5}{2(\nu_1-\nu_2)}, \ldots\ldots$$

the time interval between two successive minima.

$$= \frac{1}{\nu_1-\nu_2}.$$

Hence the time interval between two successive beats $= \frac{1}{\nu_1-\nu_2}.$

Determination of an unknown frequency – Suppose $\nu_1$ is the known frequency of tuning fork A and $\nu_2$ is the unknown frequency of tuning fork B. When the two tunning forks are sounded together, suppose they produce b beats per second than.

$$\nu_2 = \nu_1 \pm b.$$

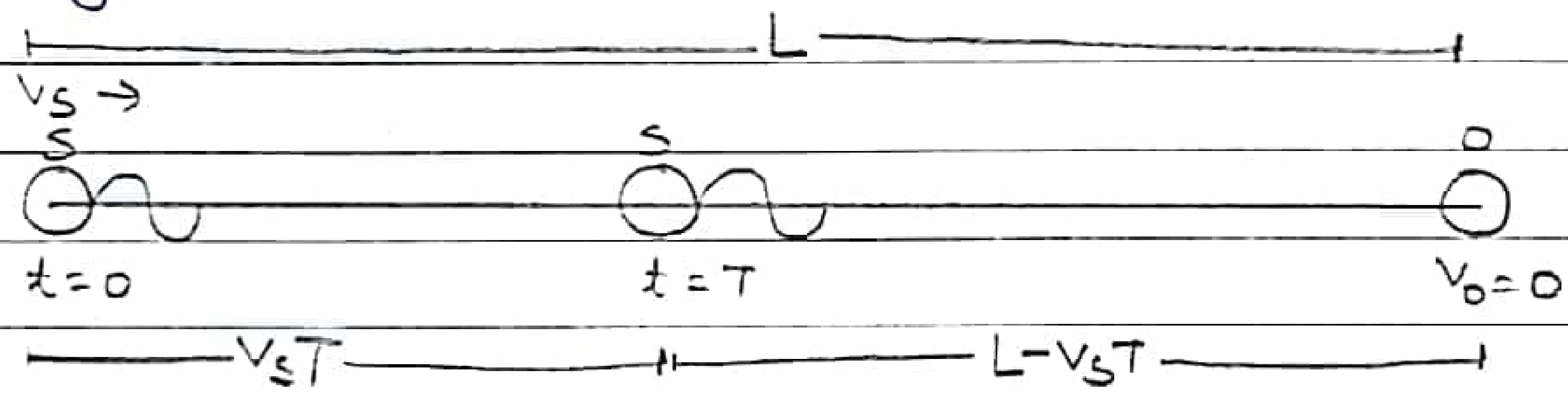
The exact frequency may be determined by two methods.

1. Loading method – Attach a little wax to the prong of the tunning fork B. Again find the number of beats per second. If the frequency of B is greater than that of A, then the attaching of a little wax lowers its frequency and reduce the difference in frequencies of A and B. This would decrease the beat frequency. On the other hand if the beat frequency is increase on loading the prong then unknown frequency is less than that of A.
So when beat frequency decrease unknown frequency is.
$$\nu_2 = \nu_1 + b.$$
When beat frequency increase unknown frequency is.
$$\nu_2 = \nu_1 - b.$$

2. Filling method – If a prong of the tuning fork B is filled its frequency increase.
If on filling the prong of B, the beat frequency decreases then $\nu_2 = \nu_1 - b$.
If on filling the prong of B, the beat frequency increases then $\nu_2 = \nu_1 + b$.

– Doppler effect – Whenever there is a relative motion between the source of sound, the observer and the medium. The frequency of sound as received by observer is different from the frequency of sound emitted by the source. The apparent change in the frequency of sound is called Doppler effect. It hold not only for sound waves but also for electromagnetic waves

Apparent frequency when the source moves towards the stationary observer.




Consider a source S moving with speed $V_S$ towards an observer O who is at rest with respect to medium. If $\nu$ is the frequency of vibration of the source, then it sends out sound wave with speed $v$ at a regular interval of $T = \frac{1}{\nu}$

At time $t = 0$ the source is at a distance L from the observer. In the mean time the source has moved a distance $V_S T$ towards the observer and is now at distance $L - V_S T$ from the observer. The second compression pulse reaches the observer at time

$$t_2 = T + \frac{L - V_S T}{v}$$

and time for first pulse.

$$t_1 = \frac{L}{v}$$

The time interval between two successive pulse as detected by the observer.

$$T' = t_2 - t_1 = T + \frac{L - V_S T}{v} - \frac{L}{v}$$

$$= \left(1 - \frac{V_S}{v}\right)T = \frac{v - V_S}{v} \times T.$$

The apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v}{(v - V_S)} \cdot \frac{1}{T}.$$

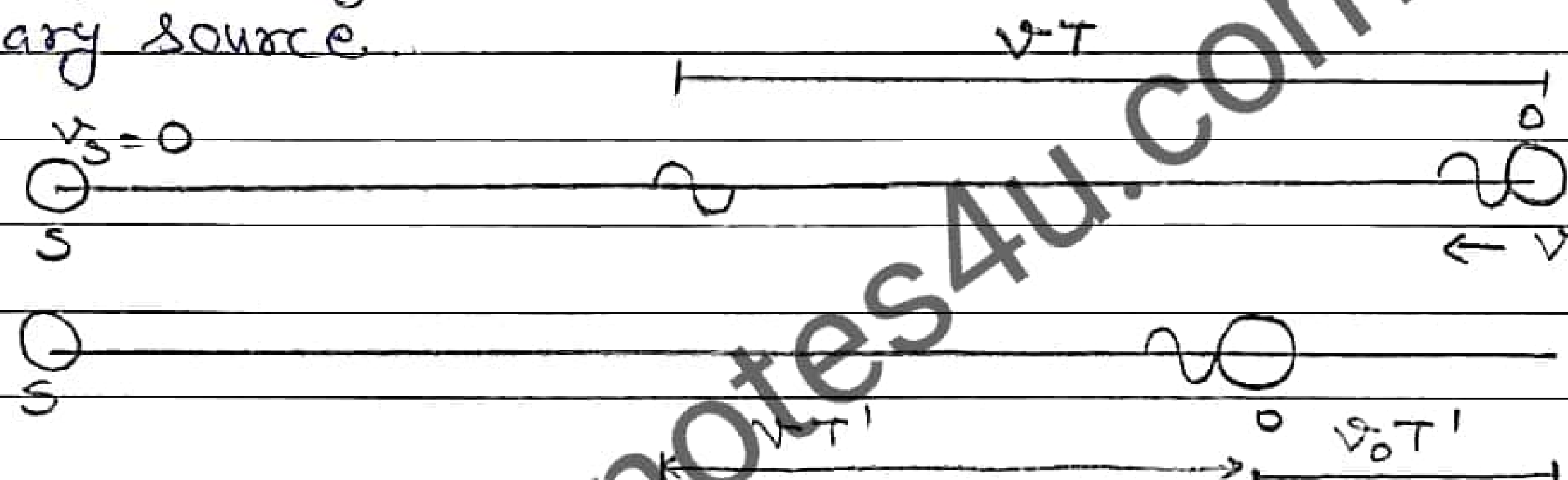
$\nu' > \nu$. Hence the pitch of sound appears to increase.

- Apparent frequency when the source moves away from the stationary observer — If the source moves away from the observer then the apparent frequency of sound can be obtained by replacing $v_s$ by $-v_s$

$$\text{then } \nu' = \frac{v}{v+v_s} \times \frac{1}{T}$$

$\nu' < \nu$, Hence the pitch of sound appears to decrease

b Apparent frequency when the observer moves towards the stationary source.




If $\nu$ is the frequency of vibration of the source then it sends out compression pulse at a regular interval of $T = \frac{1}{\nu}$ which travel with speed $v$.

At any instant the seperation between two compression pulses is $\lambda = vT$. When the observer receive a compression pulse the next pulse is a distance $vT$ away from him. This pulse moves towards the observer with speed $v$ and also the observer moves towards it with speed $v_o$, hence the observer will receive the second pulse a time $T'$ after receiving the first pulse

$$T' = \frac{vT}{v+v_o}$$

Hence the apparent frequency $\nu' = \frac{1}{T'} = \frac{v+v_o}{vT}$.

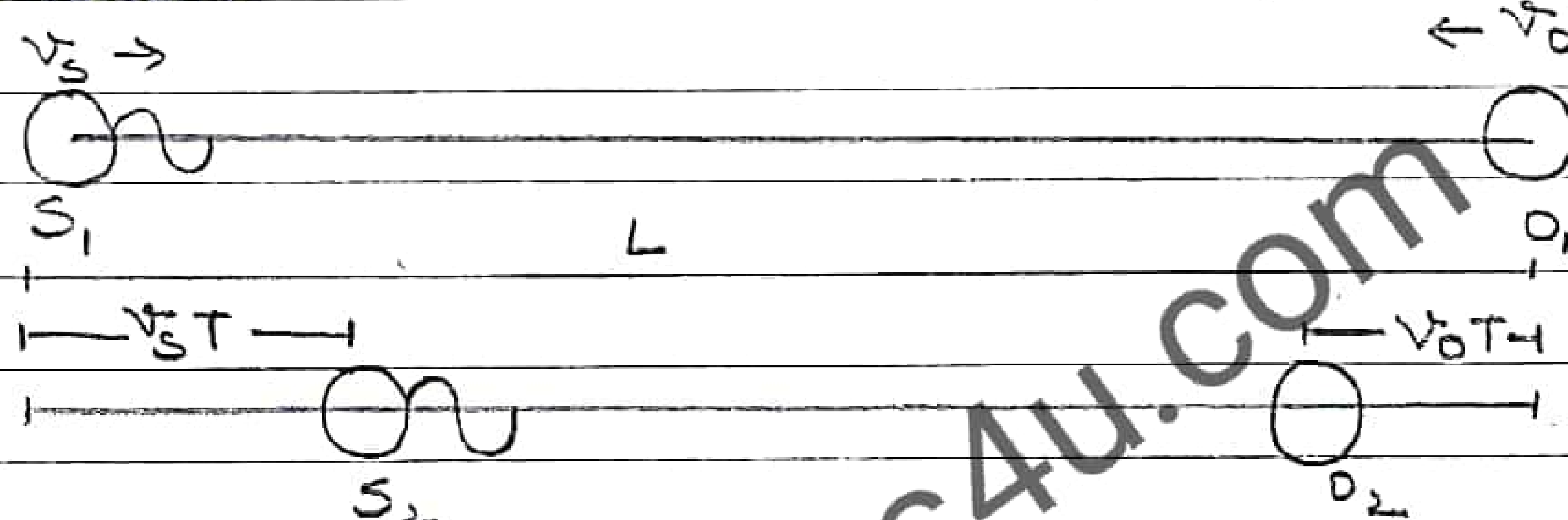
$\because \nu' > \nu$ then the pitch of sound appears to increases.

- Apparent frequency when the observer moves away from the stationary source.

$$T' = \frac{vT}{v - v_0} \quad \text{and} \quad v' = \frac{v - v_0}{vT}$$

$v' < v$ Hence the pitch of sound appears to be decreases.

c. Apparent frequency when both the source and observer are in motion.




If $v$ is the frequency of the source, it sends out compression pulses at regular interval of $T = \frac{1}{v}$

At time $t = 0$ the observer is at $O_1$ and source at $S_1$ and the distance between them is $L$. When the source emits the first pulse. Since the observer is also moving towards the source so the speed of the wave relative to the observer is $v + v_0$. Therefore the observer will receive the first pulse at time $t_1 = \frac{L}{v + v_0}$

At time $t = T$ both the source and observer moved towards each other covering distance $S_1S_2 = v_s T$ and $O_1O_2 = v_0 T$. The new distance between observer and source is.

$$S_2O_2 = L - (v_s + v_0)T.$$

The second pulse will reach the observer at time.

$$t_2 = T + \frac{L - (v_s + v_0)T}{v + v_0}$$

then the time interval $T' = t_2 - t_1$

— $T' = T + \dfrac{L - (v_s + v_o)T}{v + v_o} - \dfrac{L}{v + v_o} = \dfrac{v - v_s}{v + v_o} \times T$

then apparent frequency $\nu' = \dfrac{1}{T'} = \dfrac{v + v_o}{v - v_s} \times \dfrac{1}{T}$.

— when the source moves towards the observer and the observer moves away from the source —

the apparent frequency

$$\nu' = \frac{v - v_o}{v - v_s} \times \frac{1}{T}$$

1 If the medium also moves with a velocity $v_m$ in the direction of propagation of sound.

then $\nu' = \dfrac{v + v_m - v_o}{v + v_m - v_s} \times \dfrac{1}{T}$.

2 If the medium moves in opposite direction of sound.

then $\nu' = \dfrac{v - v_m - v_o}{v - v_m - v_s} \times \dfrac{1}{T}$.

— Doppler effect in sound is asymmetric —

Suppose a source of sound moves towards a stationary observer with speed $v'$, then the observed frequency will be $\nu' = \dfrac{v}{v - v'} \times \dfrac{1}{T}$.

Now if the observer moves towards the stationary source with the same speed $v'$ then the observed frequency will be $\nu'' = \dfrac{v + v'}{v} \times \dfrac{1}{T}$.

clearly $\nu' \neq \nu''$ Hence Doppler effect in sound is said to asymmetric, However the Doppler effect in light is symmetric.

- When the observer is at rest and the source moves with a supersonic speed, the resultant wave motion is a conical wave called a shock wave. It is due to the shock wave that we hear a sudden and violent sound called sonic boom when a supersonic jet passes by.

- No Doppler effect is observed.

I. When both the source and the observer moves in the same direction with the same speed.

II. When either the source or the observer is at the center of a circle and the other is moving along it with an uniform speed.

(III) When both the source and observer are at rest and the wind alone is blowing.

- The ratio of the speed of the source to the speed of the sound $\left(\frac{v}{v_s}\right)$ is called mach number.

- Music — The sound which has a pleasing sensation to the ears is called music.

- Noise — The sound which has non-pleasing or jarring effect on the ears is called noise.

- Characteristics of musical sound.

a. Loudness — It is the amount of energy crossing unit area around a point in one second. It is depends on.

1. Intensity is proportional to the square of the amplitude.
2. The surface area of the sounding body.
3. Density of the medium.
4. The presence of other objects around the sounding body.
5. The distance of the source from the listener.
6. The motion of air.

b. Pitch – Pitch is a sensation which helps a listener to distinguish between a high and a grave note. Pitch depends on frequency. The pitch of sound can be changed by Doppler effect.

c. Quality – Quality of sound enable us to distinguish between two sounds of same pitch and loudness. It is due to the quality of sound that one can recognise some one.

– Threshold of hearing – The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing.

– musical scale – A series of notes whose fundamental frequencies have definite ratios and which produce a pleasing effect on the ear when sounded in succession constitute a musical scale.

– Reverberation – The persistence of audible sound after the source has ceased to emit sound is called reverberation.

– Reverberation time – It is defined as the time which sound takes to fall in intensity to one millionth ($10^{-6}$) part of its original intensity after it was stopped.

– methods for controlling reverberation time.

1. Covering walls and doors with absorbent materials.
2. Providing open windows in the space.
3. Providing heavy curtains with folds and folding or opening some of the curtains.
4. Decorating the walls with pictures and maps.
5. By increasing the number of audience.
6. By the floor which help in absorbing sound.

Date:

Close organ Pipe.

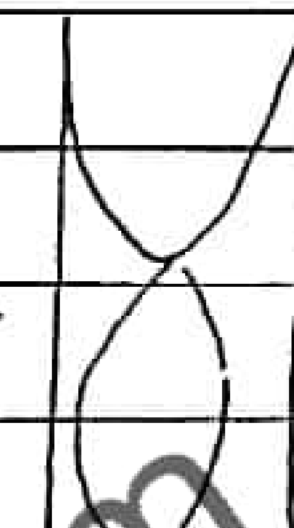
(i) First normal mode of vibration:

L.

$$L = \frac{\lambda_1}{4} \qquad \text{then } \lambda_1 = 4L.$$

$$\text{frequency } \nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}. \quad \text{—(I)}$$

(ii) Second normal mode of vibration



L

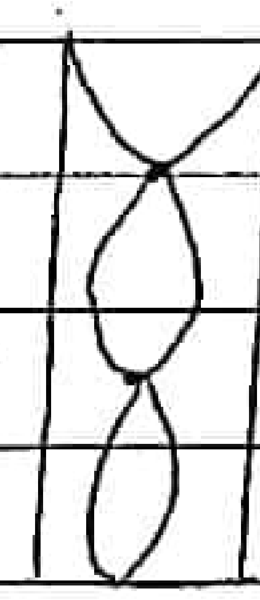
$\lambda/4$

$\lambda/2$.

$$L = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} = \frac{3\lambda_2}{4}. \quad \text{then } \lambda_2 = \frac{4L}{3}.$$

$$\text{frequency } \nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3} = \frac{3v}{4L}. \quad \text{—(II)}$$

(iii) third normal mode of vibration.



L

$\lambda/4$

$\lambda/2$

$\lambda/2$.

$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = \frac{5\lambda_3}{4} \quad \text{then } \lambda_3 = \frac{4L}{5}.$$

$$\text{frequency } \nu_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5} = \frac{5v}{4L}. \quad \text{—(III)}$$

So $\nu_2 = \frac{3v}{4L} = 3\nu_1$ this is called third harmonic and is the first overtone

$\nu_3 = \frac{5v}{4L} = 5\nu_1$ this is called fifth harmonic and is the second overtone.

then:

$\nu_n = \frac{(2n-1)v}{4L} = (2n-1)\nu_1$ this is called $(2n-1)$ harmonic and is the $(n-1)$ overtone.

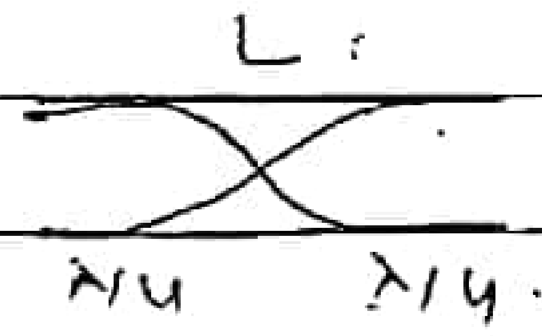
– Harmonics – these are the notes / sounds of frequency equal to or an integral multiple of fundamental frequency. Thus, First, Second, third .... harmonics have frequency $\nu, 2\nu, 3\nu$ .....

Date: / /

Overtones :- these are the notes / sounds of frequency twice / thrice / four times .... the fundamental frequency. Thus, first, second, third ... overtones have frequencies $2\nu, 3\nu, 4\nu \ldots$

— Open organ pipe.

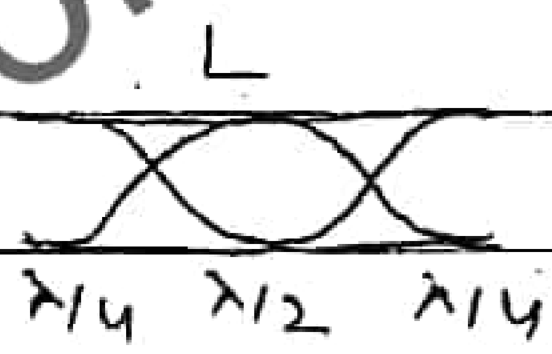
(I) First normal mode of vibration.




$$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{4} = \frac{\lambda_1}{2} \quad \text{then } \lambda_1 = 2L$$

frequency $\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ —(1)

(II) Second normal mode of vibration.

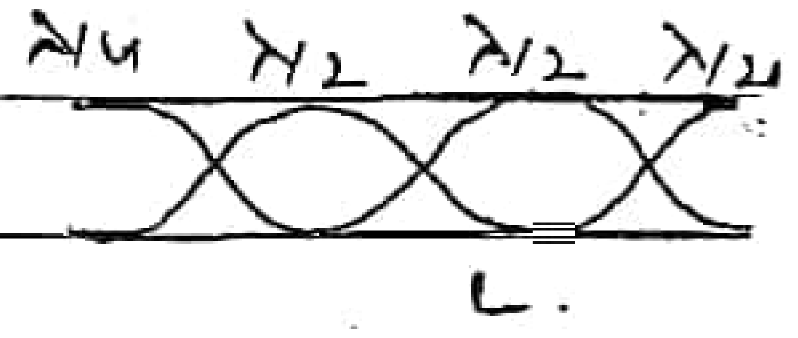



$$L = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \frac{4\lambda_2}{4} = \frac{2\lambda_2}{2} \quad \text{then } \lambda_2 = \frac{2L}{2}$$

frequency $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{2L/2} = \frac{2v}{2L} = 2\nu_1$ —(II)

This is called second harmonic and first overtone.

(III) Third normal mode of vibration.




$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4}$$

$$L = \frac{6\lambda_3}{4} \Rightarrow \text{so } \lambda_3 = \frac{4L}{6} = \frac{2L}{3}.$$

frequency $\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3} = \frac{3v}{2L} = 3\nu_1$

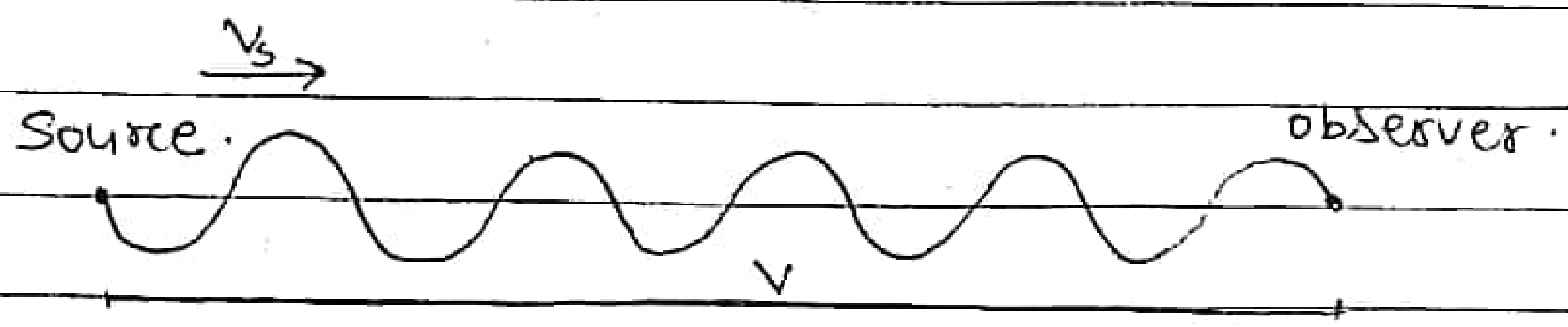
This is called third harmonic and second overtone.

then $\nu_n = n\nu_1$

This is called nth harmonic and $(n-1)$ overtone.

## Doppler effect:

Case I :- When Source is moving towards stationary observer




Let real or true frequency $N$, wavelength $\lambda$ and speed of sound $V$.

then $\lambda = \frac{V}{N}$

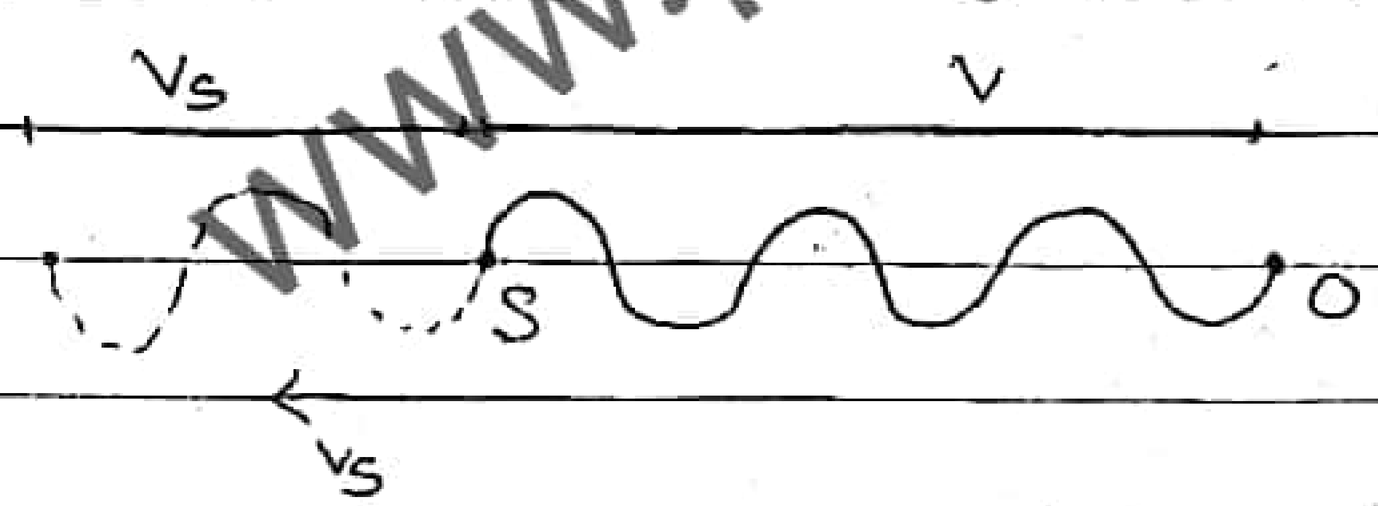
When the Source move towards the observer with speed $V_s$ then

$$\lambda' = \frac{V - V_s}{N}$$

So apparent frequency $N' = \frac{V}{\lambda'} = \left(\frac{V}{V - V_s}\right)N$

then $N' > N$. apparent frequency increase

Case II When Source is move away from observer.




$\lambda = \frac{V}{N}$ when Source and observer at rest.

$\lambda' = \frac{V + V_s}{N}$ when Source move away from observer

then apparent frequency.

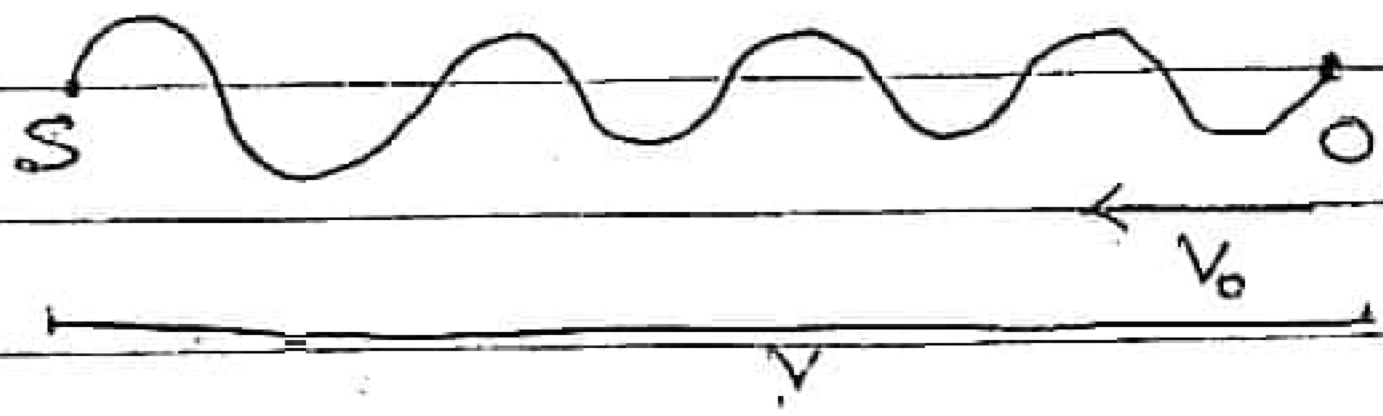
$$N' = \frac{V}{\lambda'} = \left(\frac{V}{V + V_s}\right)N.$$

clearly $N' < N$ so apparent frequency is decrease.



Teacher's Signature

Case III :- When observer move towards Stationary Source.



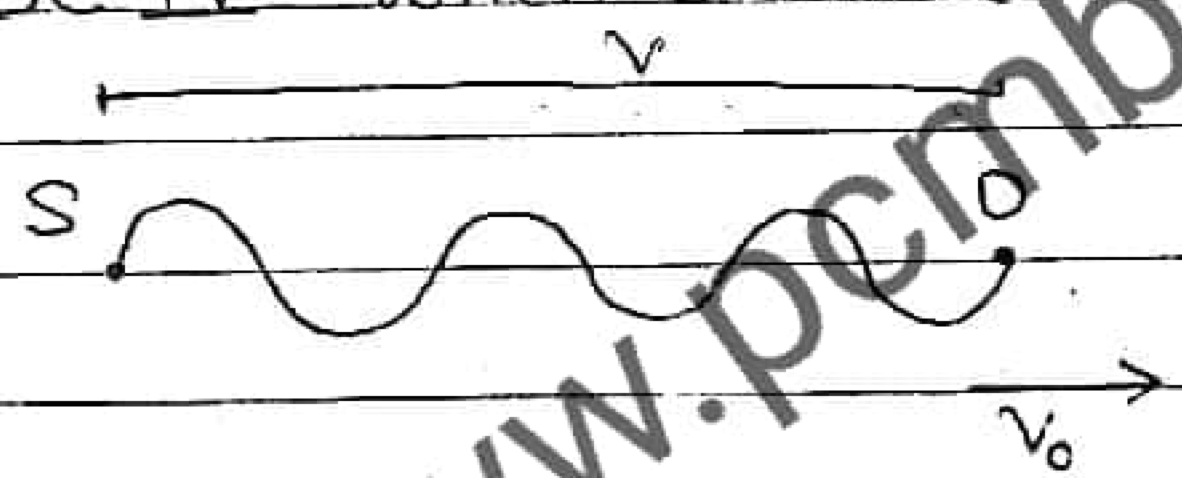

$$\lambda = \frac{V}{N} \quad \text{when both are stationary.}$$

apparent frequency $N' = \frac{V+V_0}{\lambda}$

$$N' = \frac{V+V_0}{V} \times N$$

clearly $N' > N$
so frequency increase

Case IV when observer move away from source.




$$\lambda = \frac{V}{N} \quad \text{when both stationary.}$$

apparent frequency $N' = \frac{V-V_0}{\lambda}$

$$= \frac{V-V_0}{V} \times N$$

clearly $N' < N$
So frequency decrease.



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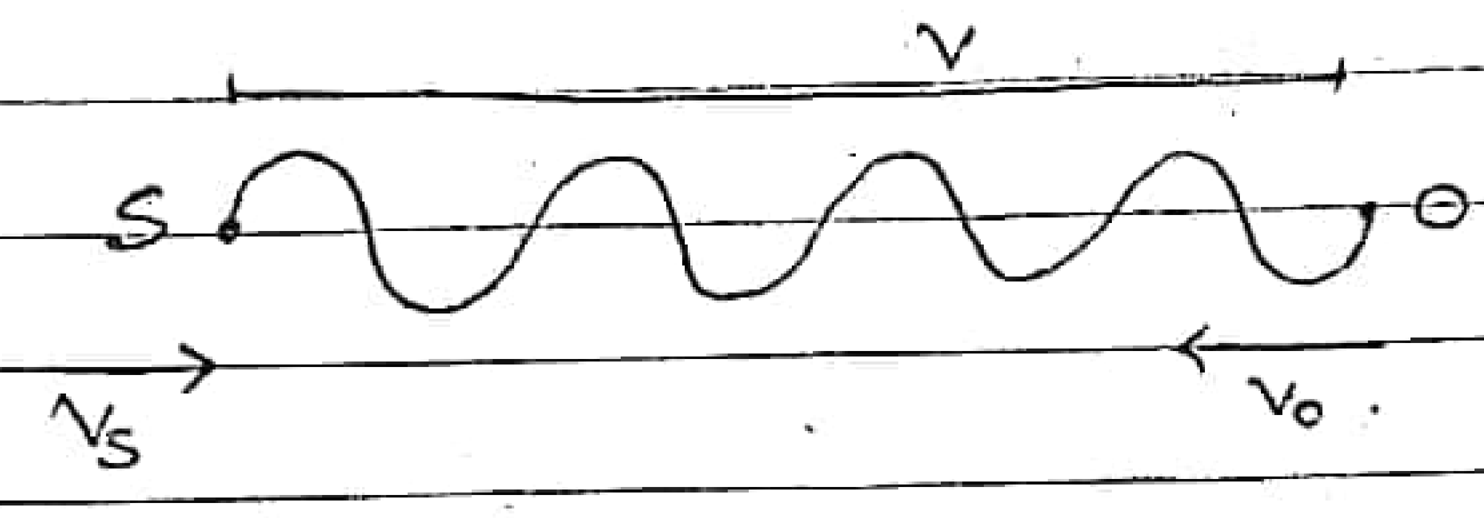
Case V :- when Source and observer move towards.



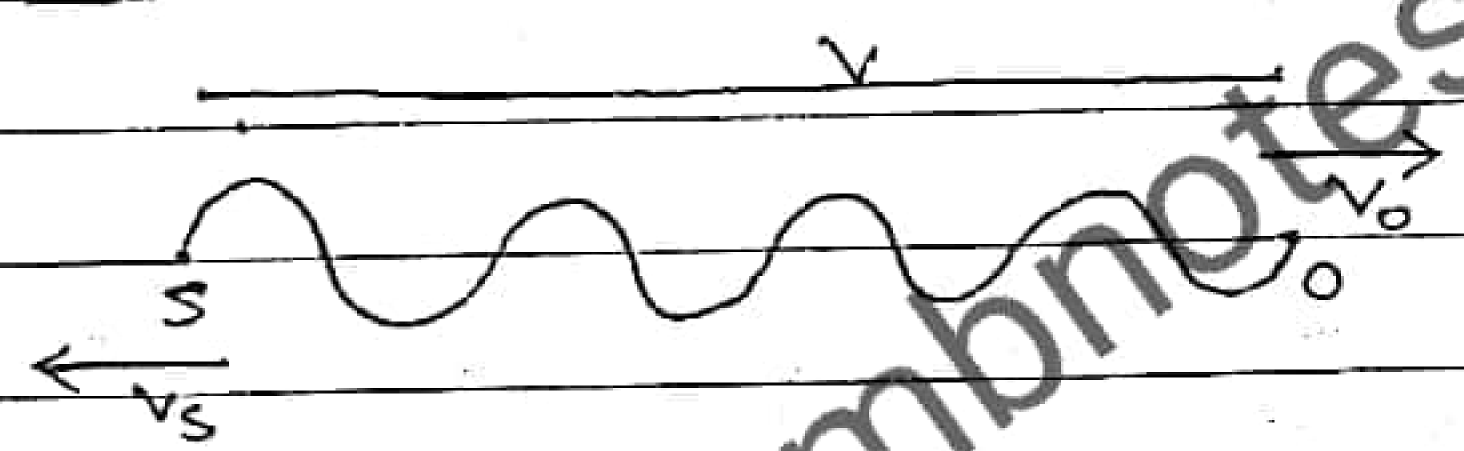

$$\lambda = \frac{V - V_s}{N}$$

apparent frequency $N' = \frac{V + V_o}{\lambda}$

$$N' = \frac{V + V_o}{V - V_s} \times N$$

clearly $N' > N$. so frequency increase.

case VI : when source and observer moves away.




$$\lambda = \frac{V + V_s}{N}$$

apparent frequency $N' = \frac{V - V_o}{\lambda}$

$$N' = \frac{V - V_o}{V + V_s} \times N$$

clearly $N' < N$. So frequency decrease.

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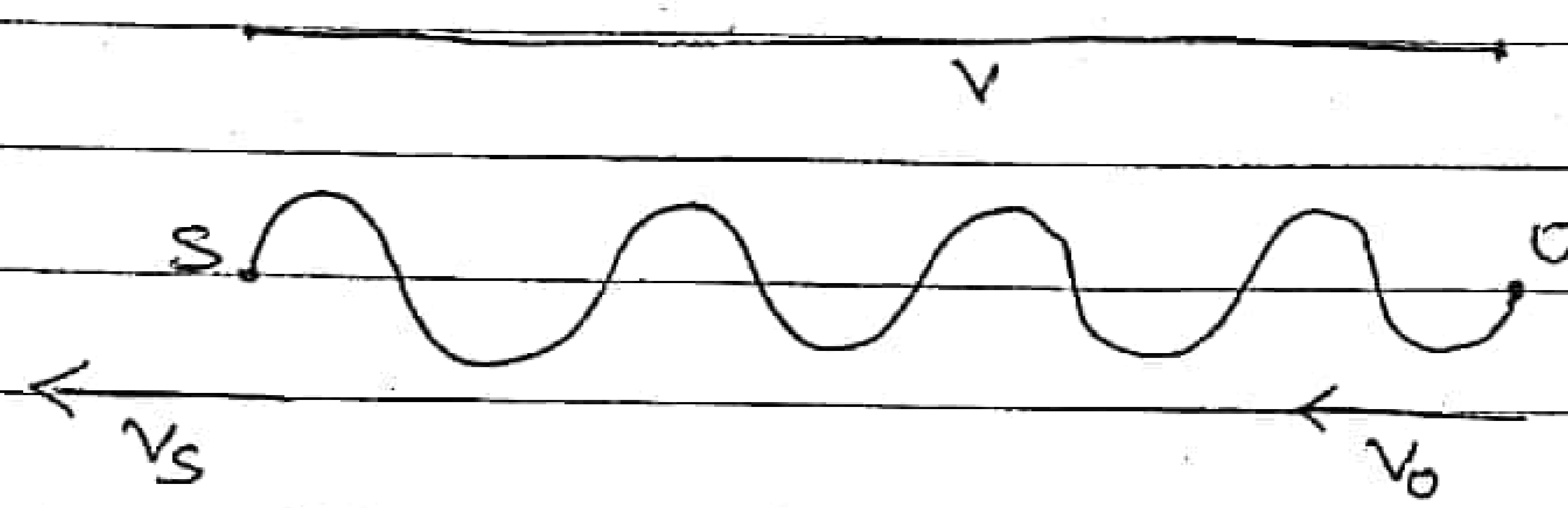
Case VII : when Source is moving away from observer and observer move toward Source.



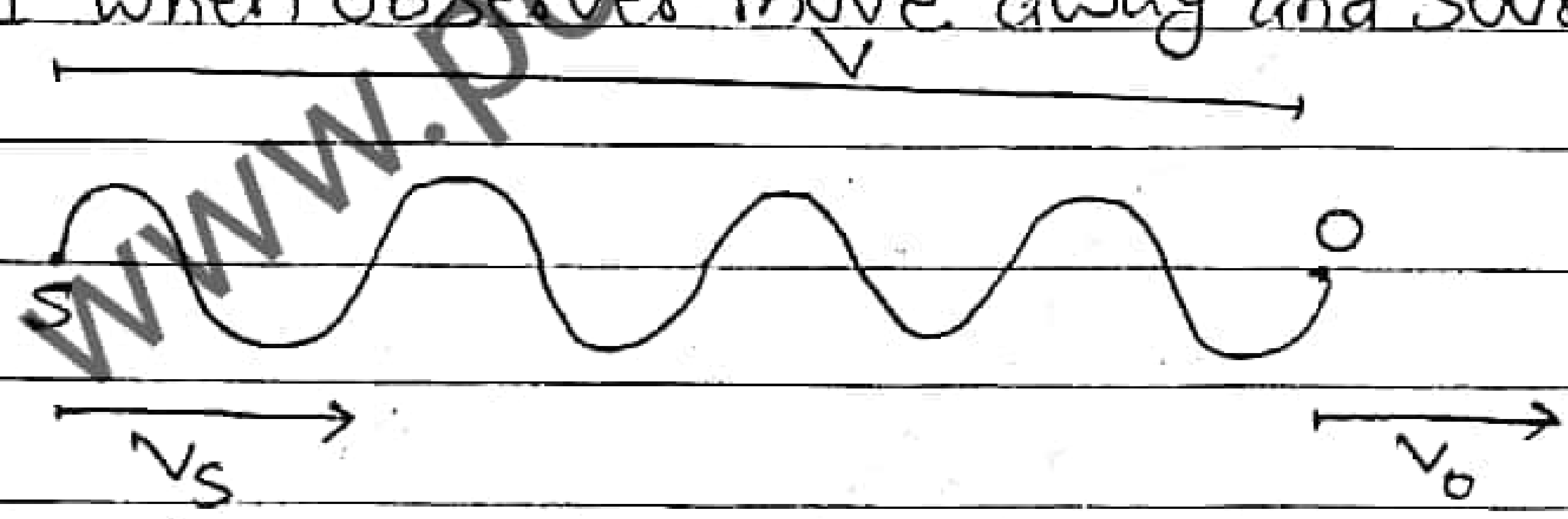

$$\lambda = \frac{V+V_s}{N}.$$

apparent frequency $N' = \frac{V-V_o}{\lambda}$.

$$N' = \frac{V-V_o}{V+V_s} \times N.$$

then Intensity of Sound Increase or decrease according to velocity

Case:-VIII when observer move away and Source move toward




$$\lambda = \frac{V-V_s}{N}.$$

apparent frequency $N' = \frac{V-V_o}{\lambda}$.

$$N' = \frac{V-V_o}{V-V_s} \times N.$$

then intensity of sound Increase or decrease according to velocity.



Teacher's Signature