Work, Energy and Power.

– Work – Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force. If the force makes angle θ with the direction of displacement s then the work done is $W = F.S.\cos\theta$.

Work done is a scalar quantity. It can be positive or negative or zero depending on θ. The work done by friction or viscous force on a moving body is negative.

(i) If $\theta = 0$ then $\cos 0 = 1$

So $W = FS$ work done is maximum.

(ii) If $\theta = 90°$ then $\cos 90 = 0$.

$W = 0$ work done is zero.

(iii) If $\theta = 180°$ then $\cos 180 = -1$

$W = -FS$ work done is negative.

– Unit work – It is the amount of work done when a unit force displaces a body through a unit distance in the direction of the force.

The Dimensional formula of work is ML^2T^{-2}

Work done in term of rectangular component.

$F = F_x i + F_y j + F_z k$

$S = S_x i + S_y j + S_z k$.

$W = F_x S_x i + F_y S_y j + F_z S_z k$.

For a varying forces the work done is equal to the definite integral of the force over the given displacement.

When the force varies both in magnitude and direction the work done is

$$W = \int_{S_1}^{S_2} F\, ds \cos\theta.$$

– Energy – Energy of a body is defined at its capacity or ability to do work. The energy of a body is measured by the amount of work done

Energy is a scalar Quantity. The diamensional formula of energy is ML^2T^{-2} and S.I. unit is Jule.

- Mechanical energy – The energy Produced by the mechanical means is called mechanical energy. It has two forms.

a. kinetic energy b. Potential energy.

a. kinetic energy – The energy Possessed by a body by virtue of its motion is called kinetic energy. A moving object can do work. The amount of work that a moving object can do before coming to rest is equal to its kinetic energy.

work done $W = FS = mas$. —(1).

$v^2 - u^2 = 2aS$. where u – initial velocity $= 0$.

$v^2 = 2aS$.

$aS = \frac{1}{2}v^2$

Put the value in eq. (1).

$W = \frac{1}{2}mv^2$ this work done appears as kinetic energy (K)

so $K = \frac{1}{2}mv^2$

- kinetic energy by calculus method – Let a body of mass m initially at rest. A force F is applied on the body to Produce a displacement ds in its own direction. The small work done is.

$$dW = \vec{F}.\vec{ds} = Fds$$

$$= ma.ds = m.\frac{dv}{dt}.ds$$

$$= m.dv.\frac{ds}{dt} = m.v.dv.$$

The total work done to increase its velocity from 0 to v

$$W = \int dW = m\int_0^v v.dv.$$

$$= m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2.$$

Relation between K.E and linear momentum.

As linear momentum $P = mv$.

and. $K.E = \frac{1}{2} mv^2$

$$= \frac{1}{2m}(m^2v^2) = \frac{P^2}{2m}.$$

then $P^2 = 2m.K.E.$

$$P = \sqrt{2m.K.E}.$$

– **Work energy theorem for a Constant force** –

The work done by the net force F acting on a body is equal to the change produced in the kinetic energy of the body.

Suppose a constant force 'F' acting on a body of mass 'm', produce acceleration 'a' in it. After covering distance 's', the velocity change from u to v.

$$v^2 - u^2 = 2as$$

multiply both side by $\frac{1}{2}m$.

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 2as \times \frac{1}{2}m.$$

$$K_f - K_i = mas = Fs = W$$

Change in K.E of the body = Work done on the body by the net force.

– **Work energy theorem for a Variable force** –

Suppose a variable force 'F' acts on a body of mass 'm' and produces displacement dx in its own direction. When the velocity of the body is v then its

$$K.E = \frac{1}{2}mv^2.$$

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}m.2v.\frac{dv}{dt} = m.v.\frac{dv}{dt}.$$

But. $m.\frac{dv}{dt} = F$

so $\frac{dK}{dt} = F.v. = F.\frac{dx}{dt}$

$$dK = F.dx$$

$$\int_{K_i}^{K_f} dK. = F\int_{x_i}^{x_f} dx.$$

$$[K]_{K_i}^{K_f} = F[x]_{x_i}^{x_f}$$

$$K_f - K_i = F(x_f - x_i) = W$$

$$K_f - K_i = W.$$

b. Potential energy - It is the energy stored in a body or a system by virtue of its position in a field or by its configuration.

Different types of Potential energy.

1. Gravitation Potential energy - It is the energy associated with the state of separation of two bodies, which attract one another through gravitational force.
2. Elastic Potential energy - It is the energy associated with the state of compression or extension of an elastic or spring objects.
3. Electrostatic Potential energy - The energy due to the interaction between two electric charges is electrostatic Potential energy.

- Gravitational Potential energy — It is Possessed by the body by virtue of its Position above the surface of the earth.

Force needed to lift the body up with zero acceleration.

$$F = mg.$$

work done on the body in raising it through height 'h'

$$W = F \times h = mgh.$$

- Conservative force - A force is Conservative if the work done by the force in displacing a Particle from one Point to another is independent of the Path followed by the Particle and depends only on the end Points.
- Non-Conservative force - If the amount of work done in moving an object against a force from one Point to another depends on the Path along which the body moves. then such a forces is Called non-Conservative force.

Principle of Conservation of mechanical energy –

If only the Conservative forces are doing work then its totel mechanical energy remains Constant.

Properties of Conservative forces.

1. A force is Conservative if it Can defined from the scaler Potential energy function $U(x)$ by the relation.

$$f(x) = -\frac{dU(x)}{dx}.$$

(ii) The work done by a conservative force on an object is Path independent and depends only on end Points.

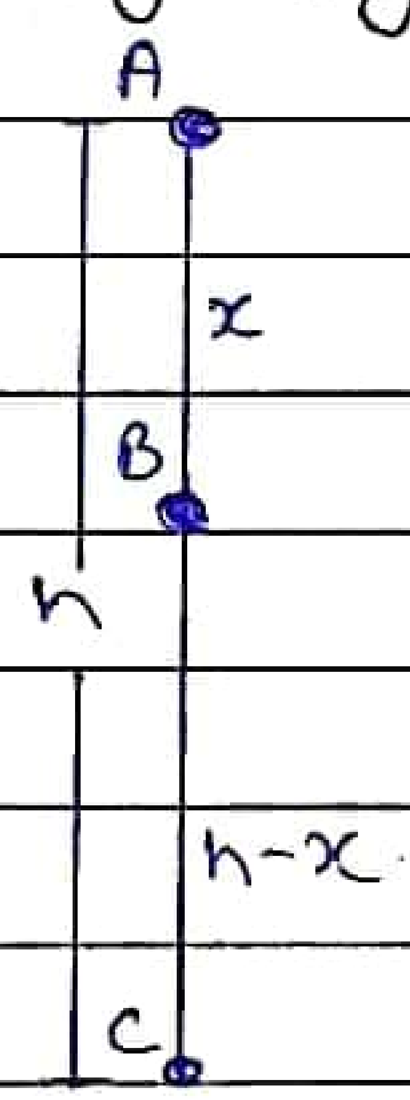
$$W = \int_{x_i}^{x_f} F(x).dx = K_f - K_i = U_i - U_f.$$

(iii) The work done by the Conservative force is zero if the object moving around any closed Path returns to its initial Position.

(iv) If only the Conservative forces are acting on body then its totel mechanical energy is Conserved.

Conservation of mechanical energy in a free falling body –

Consider a body of mass m lying at Position A at a height h above the ground. As the body falls its kinetic energy increase at the expense of Potential energy.



At Point A – The body is at rest.

$$K.E = \frac{1}{2}mv^2 = 0$$

$$P.E = mgh.$$

$$\text{Totel energy } E_A = 0 + mgh = mgh.$$

At Point B – Let the body falls freely through height x and reaches the Point B with velocity v then.

$$v^2 - u^2 = 2gs$$

$$v^2 = 2gx.$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \cdot 2gx = mgx.$$

$$P.E = mg(h-x) = mgh - mgx.$$

$$E_B = mgh - mgx + mgx = mgh.$$

At Point C - Let the body finally reaches the Point C on the ground with velocity v' then

$$v'^2 - u^2 = 2gh.$$

$$v'^2 = 2gh.$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mgh.$$

$$P.E = mg \times 0 = 0.$$

$$E_C = 0 + mgh = mgh.$$

then $E_A = E_B = E_C$

- Potential energy of a Spring - Consider an elastic spring of negligibly small mass with its one end attached to a rigid support. Its another end is attached to a block of mass m which can slide over a smooth horizontal surface. The position $x=0$ is the equilibrium position.

According to Hooke's law the spring force F_s is proportional to the displacement of the block from the equilibrium position.

$$F_s \propto x \quad \text{or} \quad F_s = -kx.$$

K is called spring constant and its S.I unit is N/m

The work done by the spring force for the small extension dx is

$$dW_s = F_s\,dx = -kx.dx.$$

$$W = \int dw = \int_{x_i}^{x_f} -kx.dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}.$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- Conservation of energy in an elastic spring - If we streached a spring to a distance x_m, its P.E is $\frac{1}{2}kx_m^2$, when it is released it begins to move under the spring force till it reaches the equilibrium position $x=0$.

At the extreme Position- Here $x = \pm x_m$ and velocity $v = 0$

$$K.E. = \frac{1}{2}mv^2 = 0$$

$$P.E = \frac{1}{2}kx_m^2 \quad \text{a maximum value}$$

At any intermediate Position - For x between $-x_m$ to $+x_m$ the energy is Partly kinetic and Partly Potential

$$\text{Total energy} = K.E. + P.E.$$

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k(x_m^2 - x^2)$$

$$v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}$$

At the equilibrium Position. Here $x = 0$.

$$P.E = \frac{1}{2}kx^2 = 0.$$

$$\text{so } \frac{1}{2}kx_m^2 = \frac{1}{2}mv^2.$$

$$v = \sqrt{\frac{k}{m}} \cdot x_m. \quad \text{maximum velocity}$$

Q. Consider a Car of mass m moving with a velocity v on a smooth road and colliding with a horizontally mounted spring of spring constant k, what is the maximum compression of the spring.

(i) when there is no friction

(ii) when there is friction and μ is friction constant.

Ans (i) Gain in P.E of spring = Loss in K.E. of the Car.

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2. \text{ so } x = \sqrt{\frac{mv^2}{k}}.$$

(ii) The work done by two opposing forces is.

$$W = -\frac{1}{2}kx^2 - \mu mgx$$

then $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx.$

Q. A ball of mass m is dropped from a height h on a platform fixed at the top of a vertical spring. The platform is depressed by a distance x.. what is the spring constant

An The ball falls through a total distance of $(h+x)$

P.E. lost by ball $= mg(h+x)$.

Work done by spring $= \frac{1}{2}kx^2$.

then By conservation of energy

$$\frac{1}{2}kx^2 = mg(h+x)$$

$$\text{so } k = \frac{2mg}{x^2} \times (x+h)$$

— Einstein's mass-energy equivalence - According to Einstein's mass-energy relation. If mass m disappears an energy $E = mc^2$ appears in some form. and when energy E disappears a mass $m = \frac{E}{c^2}$ appears.

— Principle of Conservation of energy-

(I) Energy can neither be created nor destroyed. It may be transformed from one form to another.

(II) The total energy of an isolated system remains constant.

(III) As the entire universe may be regarded as an isolated system. The total energy of the universe is constant.

— Power - The rate of doing work is called power. Power is a scalar quantity. The S.I unit of power is watt. and its diamensional formula is MLT^{-3}

$$P = \frac{W}{T} = \frac{\text{Work}}{\text{Time}}.$$

$$P = \frac{W}{t} = \frac{F.S}{t} = F.V.$$

$1\,kWh = 3.6 \times 10^6\,J$ and 1 Horse Power = 746 watt.

– Collision – A Collision is said to occur between two bodies, either if they Physically Collide against each other or if the Path of one is affacted by the force exerted by the other.

a. Elastic Collision – If there is no loss of K.E. during a Collision. it is called elastic Collision.

characteristics.

1. The momentum is Conserved.
2. Totel energy is Conserved.
3. The K.E. is Conserved.
4. Forces involved during collision are Conservative.
5. mechanical energy is not converted.

b. Inelastic Collision – If there is a loss of K.E. during Collision it is called inelastic Collision.

Characteristics.

1. The momentum is Conserved.
2. Totel energy is Conserved.
3. K.E. is not Conserved.
4. Some or all the forces involved are non-Conservative
5. A Part of mechanical energy is Converted into other form

c. Perfectly inelastic Collision – If two bodies stick together after the Collision and move as a single body with a Common velocity, then the Collision is said to be Perfectly inelastic Collision.

d. Superelastic or explosive Collision – In such a Collision there is an increase in K.E. This occur if there is a release of P.E. on an impact.

e. One-dimensional Collision – It is the Collision in which the Colliding bodies move along the straight line Path.

before and after the Collision.

f- Two dimensional collision - If two bodies do not move along the same straight line path but lie in the same plane before and after collision. the collision is said to be two dimensional collision.

- Conservation of linear momentum.
Suppose two bodies A and B collide against each other. they exert mutul impulsive force on each other during the collision.

Let F_{12} = Force exarted on body A by B.

F_{21} = Force exerted on body B by A.

According to Newtons third laws.

$$F_{12} = -F_{21}$$

$$F_{12} + F_{21} = 0$$

$$\frac{m_1(v_1-u_1)}{t} + \frac{m_2(v_2-u_2)}{t} = 0.$$

$$m_1v_1 - m_1u_1 + m_2v_2 - m_2u_2 = 0.$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2.$$

- Elastic Collision in one dimension - Consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2. Let $u_1 > u_2$. After some time the bodies collide and continue moving in the same direction. with velocities v_1 and v_2. The two bodies will separate after the collision if $v_2 > v_1$

By the Conservation of linear momentum.

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2.$$

$$m_1(v_1 - u_1) = m_2(u_2 - v_2) \quad -(1)$$

Since K.E. is also conserved in elastic collision.

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2.$$

$$m_1(v_1^2-u_1^2)=m_2(u_2^2-v_2^2) \quad -(2).$$

Divide (2) by (1).

$$\frac{m_1(v_1-u_1)(v_1+u_1)}{m_1(v_1-u_1)}=\frac{m_2(u_2-v_2)(u_2+v_2)}{m_2(u_2-v_2)}.$$

$$u_1+v_1=u_2+v_2.$$

$$u_1-u_2=v_2-v_1$$

$$\text{then } v_2=v_1+u_1-u_2 \quad -(3).$$

Putting the value of v_2 in eq. (1).

$$m_1(v_1-u_1)=m_2(u_2-v_1-u_1+u_2).$$

$$m_1v_1+m_2v_1=2m_2u_2+u_1(m_1-m_2)$$

$$v_1(m_1+m_2)=u_1(m_1-m_2)+2m_2u_2.$$

$$v_1=\frac{m_1-m_2}{m_1+m_2}u_1+\frac{2m_2u_2}{m_1+m_2}.$$

Sim. $$v_2=\frac{m_2-m_1}{m_1+m_2}u_2+\frac{2m_1u_1}{m_1+m_2}.$$

Special case

a. when $m_1=m_2=m$.

then $v_1=u_2$ and $v_2=u_1$

When two bodies of equal masses suffer one dimension elastic collision then their velocities get exchanged after collision.

b. when $m_1=m_2=m$ and $u_2=0$.

then $v_1=0$ $\quad v_2=u_1$

When an elastic body collides against another elastic body of equal mass initially at rest, then after the collision the first body comes to rest and second body moves with initial velocity of the first body.

$m_1 << m_2$ and $u_2 = 0$.

then let $m_1 = 0$.

$$v_1 = -\frac{m_2 u_1}{m_2} = -u_1 \text{ and } v_2 = 0.$$

When a light body collids against a massive body at rest the light body rebounds after the collision with an equal and opposite velocity while the massive body remains at rest.

d. When $m_1 >> m_2$ and $u_2 = 0$.

then $m_2 = 0$.

$$v_1 = \frac{m_1 u_1}{m_1} = u_1 \text{ and } v_2 = \frac{2 m_1 u_1}{m_1} = 2u_1$$

When a massive body collide against a light body at rest the velocity of the massive body remain almost unchange while the light body starts moving with twice the velocity of massive body.

— Perfectly inelastic Collision in one dimension-

A body of mass m_1 moving with velocity u_1 collide with another body of mass m_2 at rest. After the collision the two bodies move together with a common velocity. V.

then by linear momentum

$$m_1 u + m_2 \times 0 = (m_1 + m_2)V.$$

$$\text{then } V = \frac{m_1 u}{m_1 + m_2}.$$

The loss of K.E. in collision is.

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u}{m_1 + m_2}\right)^2.$$

$$= \frac{1}{2} m_1 u^2 - \frac{1}{2} \frac{m_1^2 u^2}{m_1 + m_2}.$$

$$= \frac{1}{2} m_1 u^2 \left[1 - \frac{m_1}{m_1 + m_2}\right] = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} u^2.$$

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1+m_2)v^2}{\frac{1}{2}m_1 u^2} = \frac{m_1+m_2}{m_1} \times \frac{v^2}{u^2}$$

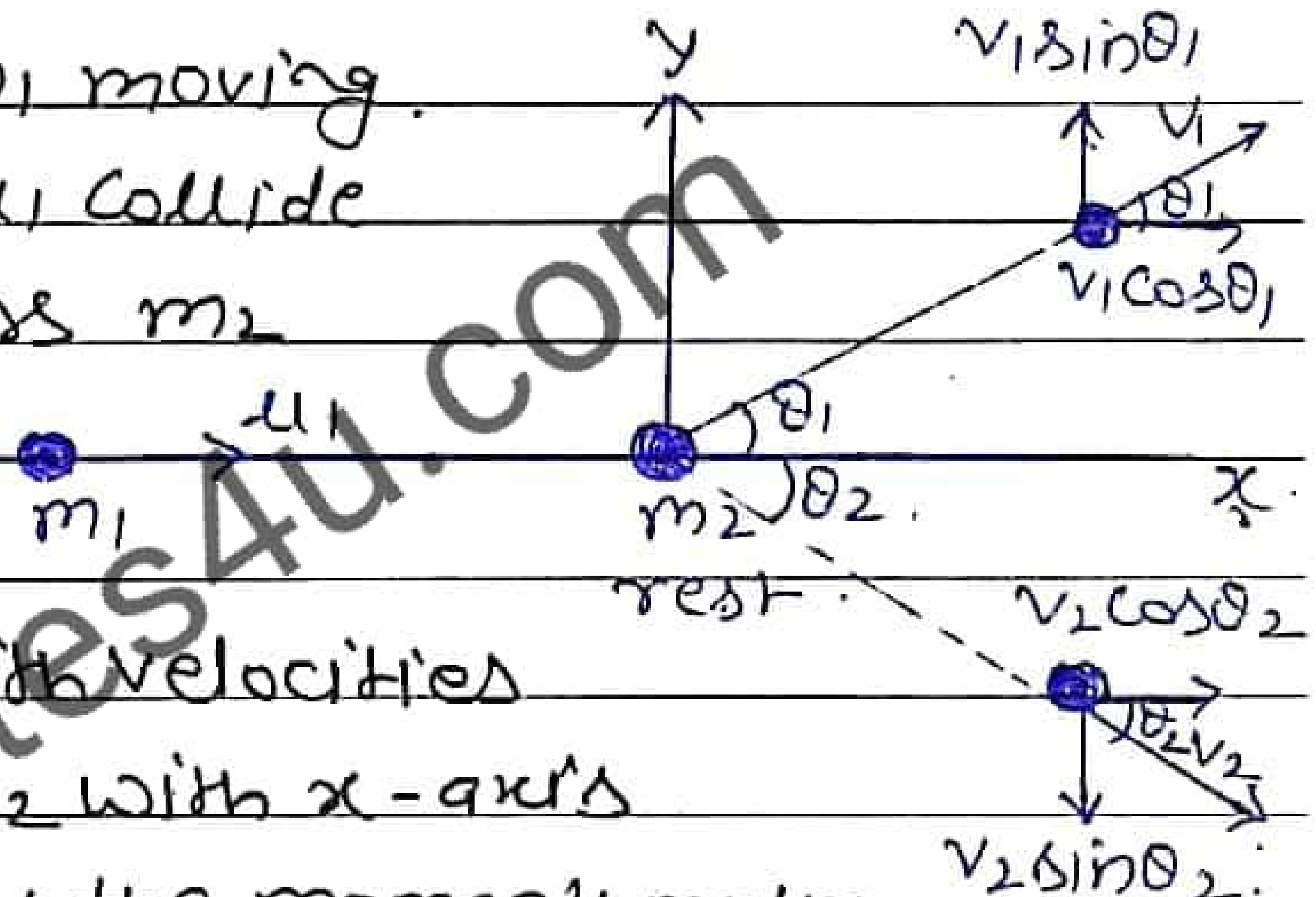
$$= \frac{m_1+m_2}{m_1}\left(\frac{m_1}{m_1+m_2}\right)^2 \frac{u^2}{u^2}.$$

$$= \frac{m_1}{m_1+m_2}.$$

If $m_2 >> m_1$ then its total energy is lost.

– Elastic Collision in two dimension

Suppose a particle of mass m_1 moving along x-Axis with velocity u_1 collide with another particle of mass m_2 at rest.



After the collision let the two particles moves with velocities v_1 and v_2 making angles θ_1, θ_2 with x-axis

After collision component of the momentum of m_1

(i) $m_1 v_1 \cos\theta_1$ + x Axis

(ii) $m_1 v_1 \sin\theta_1$ + Y Axis.

and component of the momentum of body m_2

(i) $m_2 v_2 \cos\theta_2$ + x Axis.

(ii) $m_2 v_2 \sin\theta_2$ – Y Axis.

then by linear momentum.

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{along x Axis.}$$

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad \text{along y-Axis}$$

and by K.E.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Special Case.

a. Glancing Collision – $\theta_1 = 0°$ $\theta_2 = 90°$

$u_1 = v_1$ and $v_2 = 0$.

Hence in a G.C. $K.E = \frac{1}{2} m_2 v_2^2 = 0$.

In a glancing collision the incident particle does not lose any kinetic energy and is scattered almost undeflected.

b. Head on-Collision- In such a collision the target particle moves in the direction of the incident particle

$$\theta_2 = 0$$

then $m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2$

$$0 = m_1 v_1 \sin\theta_1$$

and K.E. remain same.

c. When $m_1 = m_2 = m$.

then $\frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$

$$u_1^2 = v_1^2 + v_2^2$$

and by momentum

$$m u_1 = m v_1 + m v_2$$

so $u_1 = v_1 + v_2$.

$$u_1 . u_1 = (v_1 + v_2).(v_1 + v_2)$$

$$u_1^2 = v_1^2 + 2 v_1 . v_2 + v_2^2.$$

$$v_1^2 + v_2^2 = v_1^2 + 2 v_1 . v_2 + v_2^2.$$

$2 v_1 . v_2 = 0.$ so $\cos\theta = 0 = 90°$

then the particle move at right angle after collision.

— Cofficient of restitution :- It gives the measure of the degree of restitution of a collision and is defined as the ratio of the magnitude of relative velocity of seperation after collision to the magnitude of relative velocity of approach before collision

$$e = \frac{v_1 - v_2}{u_1 - u_2}.$$

The value of e depends upon the materials of the colliding bodies.

or a elastic Collision - $e = 1$

b. For inelastic Collision $0 < e < 1$

c. For Perfectly inelastic Collision $e = 0$.

d. For Super elastic Collision $e < 1$

Q. A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the cofficient of restitution is e. to what height it attain in nth rebounce. Totel height attain, Totel time for coming in rest.

An. $$v_2 - v_1 = e(u_1 - u_2).$$

when the ball strike the ground then $u_1 = -\sqrt{2gh}$ and $u_2 = 0$ let after Collision ball atain a height h_1 then after collision velocity of ball $v_1 = \sqrt{2gh_1}$ and $v_2 = 0$.

then $$-\sqrt{2gh_1} = e(-\sqrt{2gh}).$$

$$e = \sqrt{\frac{h_1}{h}} \quad \text{then} \quad h_1 = e^2 h.$$

in second Collision $h_2 = e^2 h_1 = e^2 . e^2 h = e^4 h$

then nth rebounce $h_n = e^{2n}.h$

Totel height till rest

$$H = h + 2h_1 + 2h_2 + \dots \infty.$$

$$= h + 2e^2h + 2e^4h + \dots$$

$$= h + 2e^2h(1 + e^2 + \dots \infty)$$

$$= h + 2e^2h\left(\frac{1}{1-e^2}\right)$$

$$= \frac{h - he^2 + 2e^2h}{1-e^2} = \frac{h + e^2h}{1-e^2} = h\left(\frac{1+e^2}{1-e^2}\right)$$

Totel time

$$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \dots \infty.$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^4h}{g}} + \dots \infty.$$

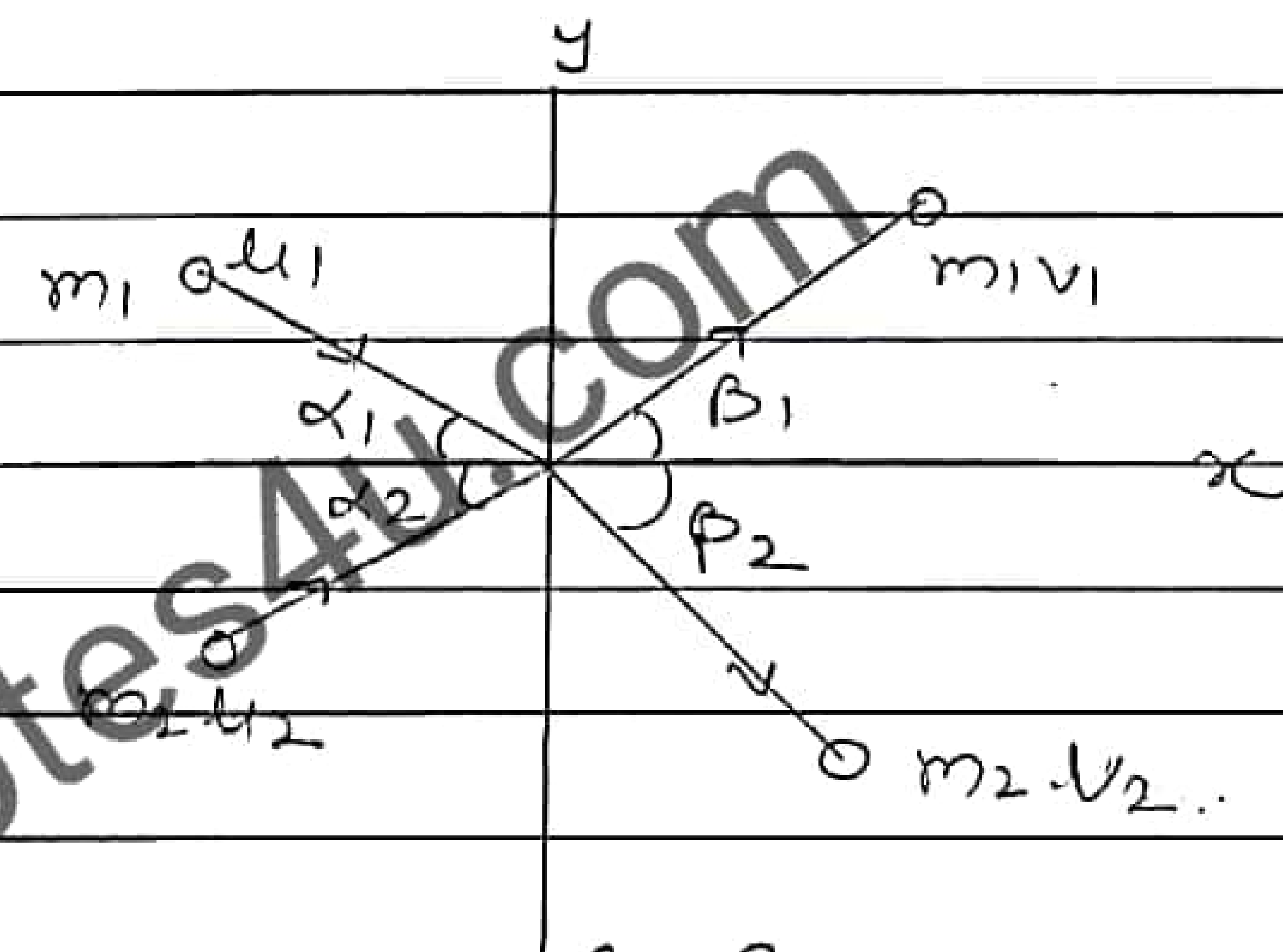
$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \cdots\infty$$

$$= \sqrt{\frac{2h}{g}}\left[1 + 2e + 2e^2 + \cdots\cdots\infty\right]$$

$$= \sqrt{\frac{2h}{g}}\left[1 + \frac{2e}{1-e}\right] = \sqrt{\frac{2h}{g}}\left[\frac{1-e+2e}{1-e}\right]$$

$$= \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$$

<u>Special.</u>



By x Axis.

$$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$$

By y-Axis.

$$-m_1u_1\sin\alpha_1 + m_2u_2 = m_1v_1\sin\beta_1 - m_2v_2\sin\beta_2.$$

$$K.E. = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$$