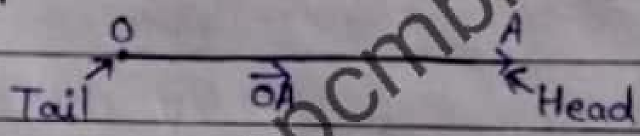
CHAPTER - 4

MOTION IN A PLANE

	Scalar	Vector
1.	It has only magnitude.	It has magnitude as well as direction.
2.	These are added according to ordinary laws of addition.	These are added according to special laws of vector addition.
3.	e.g. speed, distance, work.	velocity, displacement, force

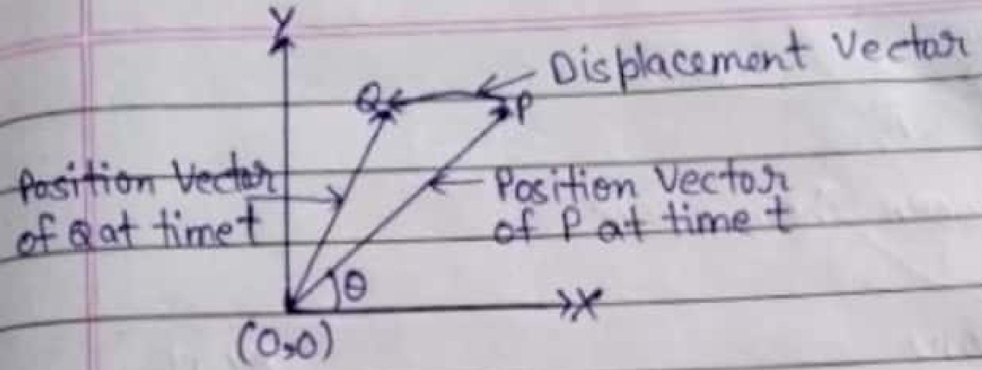
Representation of a Vector



Types of Vectors

1. Position Vector: A vector which gives position of an object with respect to the origin of a coordinate system.

2. Displacement Vector: It is that vector which tells how much & in which direction an object has changed its position in a given time interval.



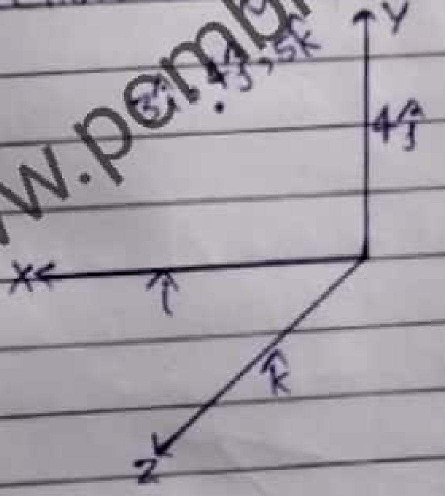
3. Equal Vectors: Same magnitude & same direction.

$|\vec{A}|$ = Magnitude of $\vec{A}$

$|\vec{B}|$ = Magnitude of $\vec{B}$

$\boxed{|\vec{A}| = |\vec{B}|}$ → Equal Vectors

4. Unit Vector: It is a vector of unit magnitude drawn in the direction of given vector.



$$\hat{i} = \frac{\vec{i}}{|\vec{i}|}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Collinear Vector

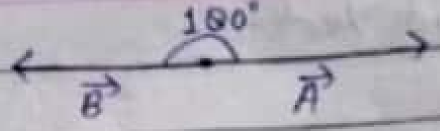
1. $\xrightarrow{\vec{A}}$ $\xrightarrow{\vec{B}}$

Connect tail to head

$\xrightarrow{\vec{A}}$ $\theta = 0°$ $\xrightarrow{\vec{B}}$

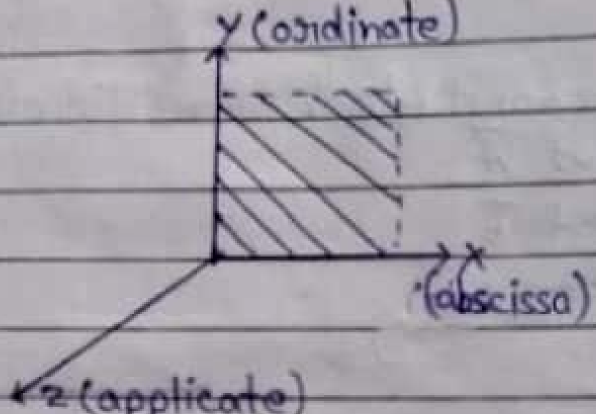
Angle between them = 0°

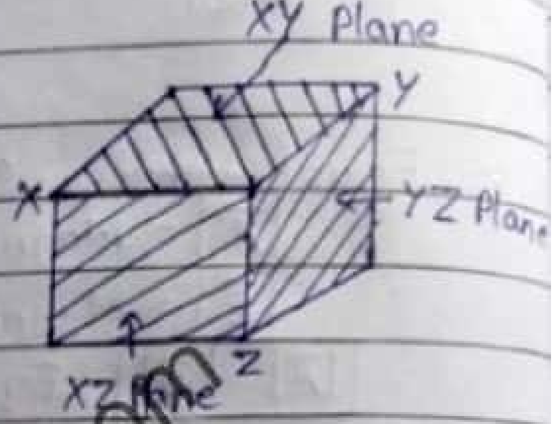
2.



Angle between them = 180°

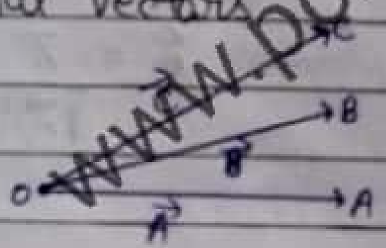
Coplanar Vectors



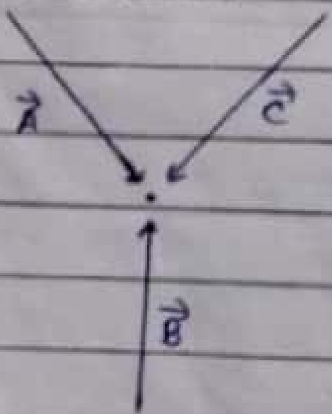


Which acts in the same plane are called coplanar vector.

Coinitial Vectors



Coterminus Vectors



Zero Vector or Null Vector

Its magnitude is zero & has an arbitrary direction.

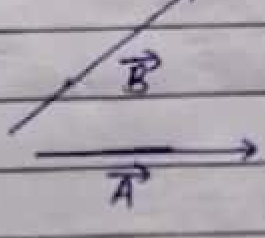
$\vec{A} + \vec{0} = \vec{A}$, $\lambda\vec{0} = \vec{0}$, $0.\vec{A} = 0$

$\lambda\vec{A} = \mu\vec{B}$ (It holds good only when A & B are zero vectors)

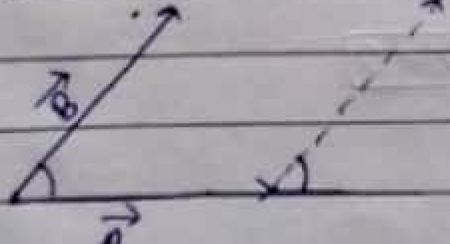
Multiplication of a vector by a Real Number

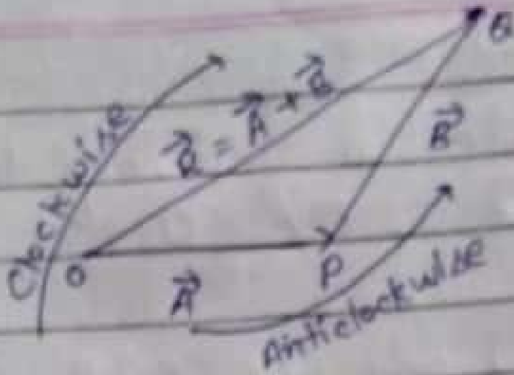
Triangle Law of Vector Addition

If two vectors can be represented both in magnitude & direction by the two sides of a Δ taken in the same order, then their resultant is represented completely both in magnitude & direction by the third side of the Δ taken in opposite order.



Connect tail to tail

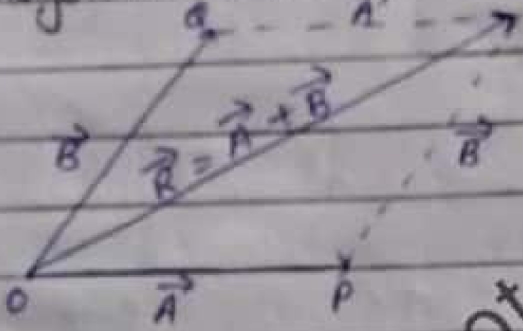




$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\boxed{\vec{A} + \vec{B} = \vec{R}}$ → Δ Law of Vector Addition

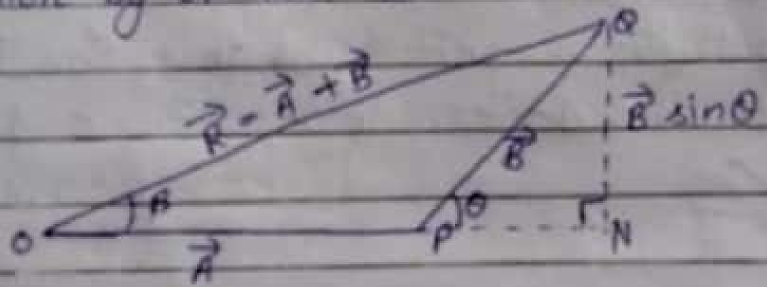
Parallelogram Law of Vector Addition



$\boxed{\vec{R} = \vec{A} + \vec{B}}$ → ||gm law of Vector Addition

Analytical Treatment of the Δ Law of Vector Addition

Let $\vec{A}$ & $\vec{B}$ be represented both in magnitude & direction by $\vec{OP}$ & $\vec{PQ}$ in ΔOPQ in same order.



$\cos\theta = \frac{PN}{PQ} = \frac{PN}{B} \Rightarrow \boxed{PN = B\cos\theta}$

$\sin\theta = \frac{QN}{PQ} = \frac{QN}{B} \Rightarrow \boxed{QN = B\sin\theta}$

In ΔQNO

$$H^2 = B^2 + P^2$$

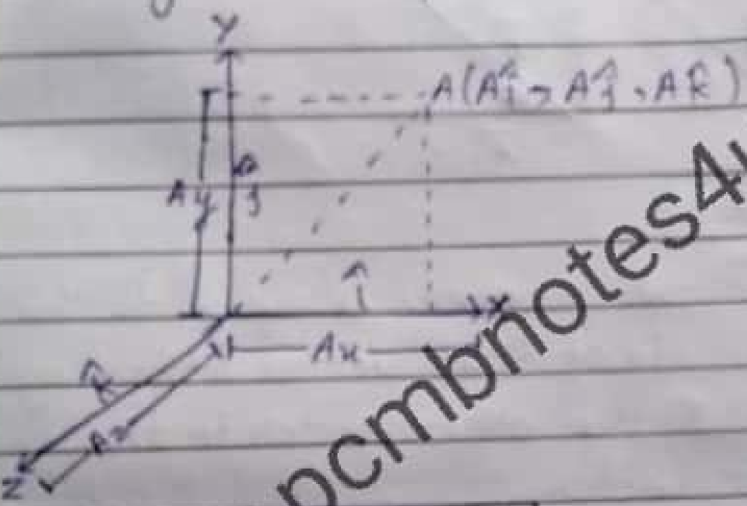
$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{A + B + 2AB\cos\theta}}$$ ⇒ Analytical Treatment of the Δ law of vector addition

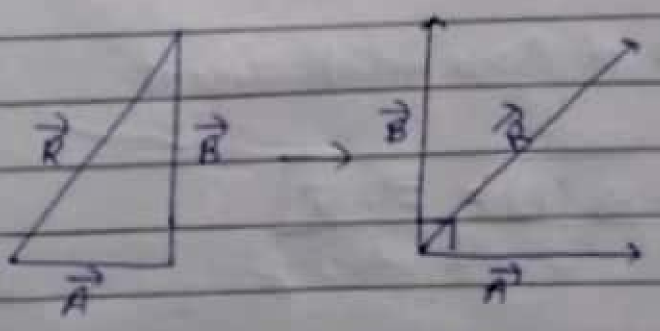
Rectangular Component of a Vector



$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$$

Direction of $\vec{R}$

$$\boxed{\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}}$$ [From fig. of Analytical Treatment - - -]



$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ}$$ [Analytical Treatment - - -]

$$\boxed{R = \sqrt{A^2 + B^2}}$$

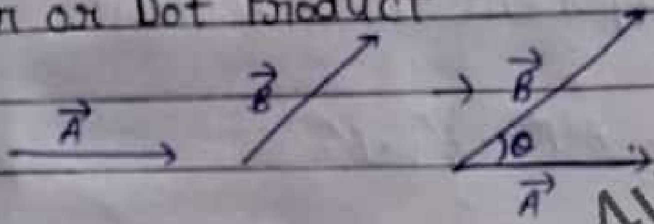
Properties of Addition of Vectors

1. $\vec{A} + \vec{B} = \vec{B} + \vec{A} \Rightarrow$ Commutative Property

2. $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C} \Rightarrow$ Associative Property

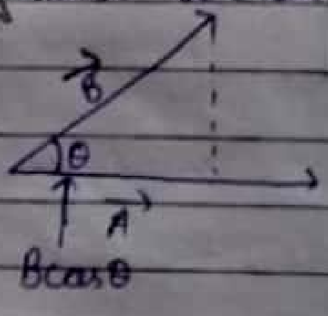
Multiplication of Vectors

1. Scalar or Dot Product



$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}| . |\vec{B}| . \cos\theta}$$

Scalar product of two numbers is equal to the product of magnitude of first vector & the magnitude of the component of the other vector in the direction of first vector.



e.g. $\boxed{Work = \vec{F} \cdot \vec{S}}$
↳ Dot product of force & displacement.

$\boxed{\text{Instantaneous Power } (P) = \vec{F} \cdot \vec{v}}$
↳ Dot product of force & velocity.

<u>Properties</u>

1. Commutative Property:

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

2. Distributive over Addition:

$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

3. $\vec{A}$ & $\vec{B}$ are perpendicular

$\vec{A} \cdot \vec{B} = 0$

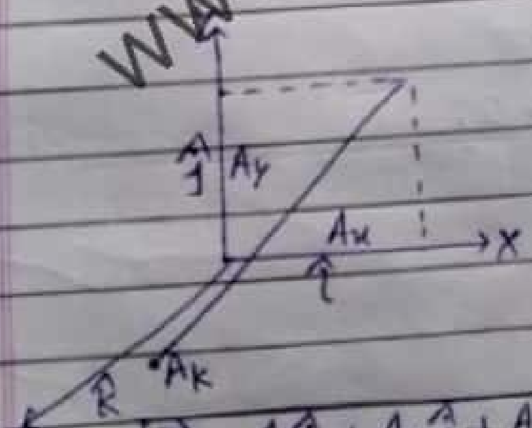
4. If $\theta = 0°$

$\vec{A} + \vec{B} = AB$

5. If $\theta = 180°$

$\vec{A} + \vec{B} = -AB$ $(\because \cos 180° = -1)$

6.



$\hat{i} \times \hat{k} = -\hat{j}$

$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$

$\hat{i} \cdot \hat{j} = |\hat{i}| . |\hat{j}| . \cos 90°$

$= 1 . 1 . 0$

$\boxed{\hat{i} \cdot \hat{j} = 0}$

Also, $\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$

$$\boxed{|R| = \sqrt{Ax^2 + Ay^2 + Az^2}}$$

$$\hat{i}\cdot\hat{i} = |\hat{i}|.|\hat{i}|.\cos\theta$$
$$= 1.1.\cos 0$$
$$= 1.1.1$$
$$\boxed{\hat{i}.\hat{i} = 1}$$

Also, $\boxed{\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1}$

Q) $\vec{F} = 3\hat{i} + 2\hat{j} + \hat{k}$
$\vec{s} = \hat{i} + 3\hat{i} + 2\hat{k}$
Find work.

Ans. Work $= \vec{F}.\vec{s}$
$= 3+6+2$
$=$

Q) $\vec{F} = 2\hat{j} - 3\hat{i} + 4\hat{k} \Rightarrow -3\hat{i} + 2\hat{j}$
$\vec{v} = 2\hat{i} + 3\hat{k}$
Find Instantaneous Power.

Ans. Instantaneous power $= \vec{F} \times \vec{v}$
$= (-6+12)$
$= 6$ watt.

Q) $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$
Find $\vec{A}.\vec{B}$ & the angle b/w these two vectors

Ans $\vec{A}\cdot\vec{B} = 2+6+12$
$= 20$

$\Rightarrow \pi^c = 180^\circ$

We know,
$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$ — (i)

$\Rightarrow \frac{\pi^c}{2} = 90^\circ$

Now,
$|\vec{A}| = \sqrt{2^2+3^2+4^2}$
$= \sqrt{4+9+16}$
$= \sqrt{29}$

$\Rightarrow \frac{\pi^c}{3} = 60^\circ$

& $|\vec{B}| = \sqrt{1^2+2^2+3^2}$
$= \sqrt{1+4+9}$
$= \sqrt{14}$

$\Rightarrow \frac{\pi^c}{4} = 45^\circ$

$\vec{A}\cdot\vec{B} = 20$

Putting $|\vec{A}| = \sqrt{29}$ & $|\vec{B}| = \sqrt{14}$ in (i)

$\Rightarrow 2\pi^c = 360^\circ$

$20 = \sqrt{29}.\sqrt{14}\cos\theta$

$\cos\theta = \dfrac{20}{\sqrt{29}.\sqrt{14}}$

$\theta = \cos^{-1}\dfrac{20}{\sqrt{29}.\sqrt{14}}$

Q] If $\theta = \cos^{-1}\left(\frac{1}{2}\right)$. find value of θ.

Soln We have,

$\theta = \cos^{-1}\left(\dfrac{1}{2}\right)$

$\cos\theta = \dfrac{1}{2}$

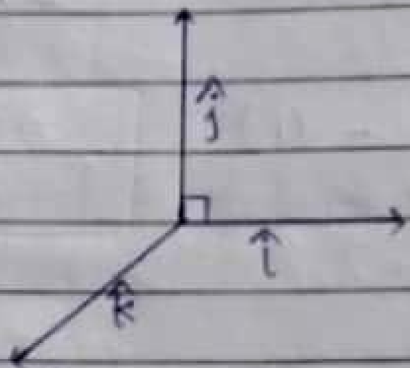
$\cos\theta = \cos 60^\circ$

$\boxed{\theta = 60^\circ}$

2. Vector Product or Scalar Product

$|\vec{A}|.|\vec{B}| = |\vec{A}|.|\vec{B}|.\sin\theta\,\hat{n}$

↳ Unit vector which is ⊥ to A & B.



$\hat{i}\times\hat{j} = |\hat{i}|.|\hat{j}|\sin\theta\,\hat{n}$

$= |1|.|1|.\sin 90^\circ\,\hat{k}$

$= 1.1.1.\hat{k}$

$\boxed{\hat{i}\cdot\hat{j} = \hat{k}}$

$\boxed{\hat{j}\cdot\hat{i} = -\hat{k}}$

$\boxed{\hat{i}\times\hat{i} = \hat{j}\times\hat{j}\times\hat{k}.\hat{k} = 0}$

3. Distributive Property

$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

Q) $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

Find $\vec{A}\times\vec{B}$.

Sol → $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix}$

$= \hat{i}(9-8) + j(4-6) + \hat{k}(4-3)$

$= \hat{i} - 2\hat{j} + \hat{k}$

Q) $\vec{A} = -2\hat{j} + 3\hat{i} \Rightarrow 3\hat{i} - 2\hat{j}$

$\vec{B} = \hat{i} - 3\hat{k} + \hat{j} \Rightarrow \hat{i} + \hat{j} - 3\hat{k}$

Find $\vec{A} \times \vec{B}$.

Ans $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & \\ 1 & 1 & -3 \end{vmatrix}$

$= \hat{i}(6-0) + \hat{j}(0+9) + \hat{k}(3+2)$

$= 6\hat{i} + 9\hat{j} + 5\hat{k}$

Q) $\vec{r} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{F} = 2\hat{j} - 4\hat{k} + \hat{i} \Rightarrow \hat{i} + 2\hat{j} - 4\hat{k}$

Find Torque.

Ans Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & -2 \\ 1 & 2 & -4 \end{vmatrix}$

$= \hat{i}(-16+4) + \hat{j}(-2+12) + \hat{k}(6-4)$

$= -12\hat{i} + 10\hat{j} + 2\hat{k}$

Q) $\vec{F} = 3\hat{i} - 4\hat{k}$

$\vec{s} = 2\hat{k} + 6\hat{i} - 3\hat{j} \Rightarrow 6\hat{i} - 3\hat{j} + 2\hat{k}$

(i) Find work done

(ii) $\angle$ b/w $\vec{F}$ & $\vec{s}$

Ans (i) Work done $= \vec{F} \cdot \vec{s}$

$= 18 - 8$

$= 10\,J$

(ii) $\vec{F} \cdot \vec{s} = |\vec{F}| . |\vec{s}| \cos\theta$

$10 = \sqrt{25} . \sqrt{49} \cos\theta$

$10 = 35 \cos\theta$

$\cos\theta = \frac{10}{35}$

$\cos\theta = \frac{2}{7}$

$$\boxed{\theta = \frac{2}{7}\cos^{-1}}$$

Q) $\vec{A} = 3\hat{i} - \hat{j} + 2\hat{k}$

$\vec{B} = 2\hat{j} - \hat{k} + 4\hat{i} \Rightarrow 4\hat{i} + 2\hat{j} - \hat{k}$

(i) Find $\vec{A}.\vec{B}$

(ii) Find $\vec{A}\times\vec{B}$

(iii) Find $\angle$ b/w A & B.

Ans-(i) $\vec{A}.\vec{B} = 12 - 2 - 2$

$= 8$ units

(ii) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 4 & 2 & -1 \end{vmatrix}$

$= \hat{i}(1-4) + \hat{j}(8+3) + \hat{k}(6+4)$

$= -3\hat{i} + 11\hat{j} + 10\hat{k}$

(iii) $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta$

$8 = \sqrt{14}.\sqrt{21}.\cos\theta$

$\cos\theta = \frac{8}{\sqrt{14}.\sqrt{21}}$

$\theta = \frac{8}{\sqrt{14}.\sqrt{21}}\cos^{-1}$

$\theta = \frac{8}{\sqrt{7}.\sqrt{2}.\sqrt{7}.\sqrt{3}}\cos^{-1}$

$\theta = \frac{8}{7\sqrt{6}}\cos^{-1}$

$$\theta = \cos^{-1} \times \frac{8}{7\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

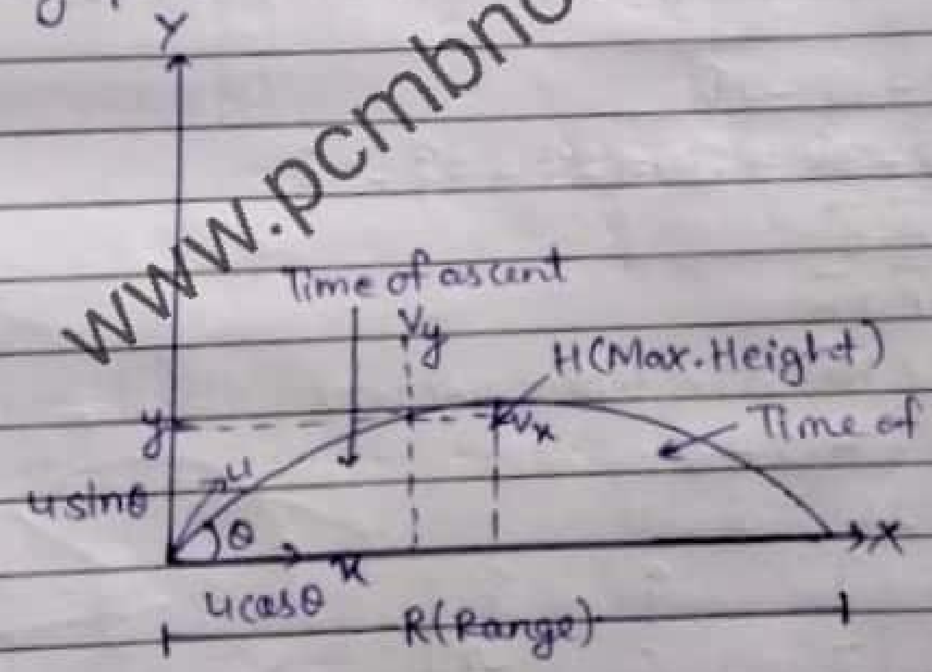
$$= \cos^{-1}\left(\frac{8\sqrt{6}}{42}\right)$$

$$= \cos^{-1}\left(\frac{4\sqrt{6}}{21}\right)$$

Projectile Motion

A projectile is a name given to any body which ones thrown into space with some initial velocity moves thereafter under the gravity alone.

The path followed by a projectile is known as trajectory path.



Time taken = Time of flight = T_f

$u\cos\theta$: Remains constant
$u\sin\theta$: Vary with time (due to gravity)
Path followed: Parabolic path

Equation of Trajectory (For horizontal motion)

$x = (u\cos\theta) \times t$ [x = horizontal velocity $\times$ Time]

$$\boxed{t = \frac{x}{u\cos\theta}}$$

Equation of Parabola (for vertical motion)

$u = u\sin\theta$, $a = -g$

$s = ut + \frac{1}{2}at^2$

$$y = u\sin\theta \times \frac{x}{u\cos\theta} + \frac{1}{2}(-g)\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

$$\boxed{y = px - qx^2}$$ Equation of parabola

Q) $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\vec{B} = 2\hat{j} - 3\hat{k} + 2\hat{i} \Rightarrow 2\hat{i} + 2\hat{j} - 3\hat{k}$

Find (i) $\vec{A}\cdot\vec{B}$

(ii) $\vec{A} \times \vec{B}$

(iii) $\angle$ b/w the two vectors

Ans (i) $\vec{A}\cdot\vec{B} = 6 + 12 + 6$

$= 24$

(ii) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & -2 \\ 2 & 2 & -3 \end{vmatrix}$

$= \hat{i}(-18 + 4) + \hat{j}(-4 + 9) + \hat{k}(6 - 12)$

$= -14\hat{i} + 5\hat{j} - 6\hat{k}$

(iii) $\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cos\theta$

$$24 = \sqrt{49}\cdot\sqrt{17}\cos\theta$$

$$24 = 7\cdot\sqrt{17}\cos\theta$$

$$\cos\theta = \frac{24}{7\cdot\sqrt{17}}$$

$$\theta = \cos^{-1}\left(\frac{24}{7\cdot\sqrt{17}}\right)$$

Q) $\vec{r} = 2\hat{j} - 3\hat{i} + \hat{k} \Rightarrow -3\hat{i} + 2\hat{j} + \hat{k}$

$\vec{F} = \hat{i} - \hat{j} + 3\hat{k}$

Find torque.

Ans Torque $(\tau) = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & -1 & 3 \end{vmatrix}$$

$$= \hat{i}(6+1) + \hat{j}(1+9) + \hat{k}(3-2)$$

$$= 7\hat{i} + 10\hat{j} + \hat{k}$$

Time of Flight (T_f):

Net vertical distance, $s = ut + \frac{1}{2}at^2$

$$0 = u\sin\theta \cdot T_f + \frac{1}{2}(-g)(T_f)^2$$

$$u\sin\theta \cdot \cancel{T_f} = \frac{1}{2}g(T_f)^2$$

$$u\sin\theta = \frac{1}{2}g \cdot T_f$$

$$\boxed{T_f = \frac{2u\sin\theta}{g}}$$

Max. Height of Projectile:
It is the max. vertical distance attend by the projectile above the horizontal plane.

$$v^2 = u^2 + 2as$$

$$0 = u^2\sin^2\theta - 2gH$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

Horizontal Range:
It is the horizontal distance travelled by the projectile during its time of flight.

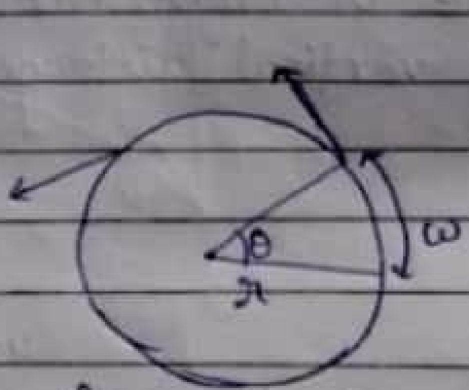
$$R = \text{Horizontal Velocity} \times \text{Time of flight}$$

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

Uniform Circular Motion
θ = angular displacement
ω = angular velocity
α = angular acceleration



Uniform Circular Motion: In uniform circular motion, the speed of the body remains same but direction of motion changes at every point.

Angular Displacement: It is the angle swept by its radius vector in the given time interval.

$$\Delta\theta = \frac{arc}{radius} \Rightarrow \boxed{\Delta\theta = \frac{\Delta s}{r}}$$

SI unit: Radian

Dimension: $[M^0L^0T^0]$

Angular Velocity: Rate of change of angular displacem

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

SI unit: Radian/sec

Dimension: $[T^{-1}]$

We know, $\omega = 2\pi\nu$ & $\nu = \frac{1}{T}$

$$\boxed{\omega = \frac{2\pi}{T}}$$

Relation b/w ω, v & r

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{\Delta t \times r}$$

$\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t \times r}$$

$$\frac{d\theta}{dt} = \frac{ds}{dt \times r}$$

$$\omega = \frac{v}{r}$$

$$\boxed{v = \omega \times r} \Rightarrow \boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

Relation b/w a, r & v

$$v = \omega \times r$$

$$\frac{dv}{dt} = \frac{d\omega \times r}{dt}$$

$$\boxed{a = r\alpha} \quad \left(\because \alpha = \frac{d\omega}{dt}\right)$$

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$$

Q) A football player kicks a ball at angle of 30° with an initial speed of 20m/s assuming that the ball travels in a vertical plane. Calculate

(a) The time at which the ball reaches the max. height

(b) Max. Height (H).

(c) Range (R).

(d) Time of flight (T_f) ($g = 10 m/s^2$)

Ans(a) $T_f = \frac{2u\sin\theta}{g}$

$$= \frac{2 \times \cancel{20}^{2} \times 1}{1\cancel{10} \times \cancel{2}_1}$$

$$= 2s$$

(b) $H = \frac{u^2\sin^2\theta}{2g}$

$$= \frac{(20)^2 \times \left(\frac{1}{2}\right)^2}{2 \times 10}$$

$$= \frac{\cancel{400}^{100} \times \frac{1}{\cancel{4}_1}}{20}$$

$$= 5m$$

(c) $R = \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(20)^2 \times \sin 60}{10}$$

$$= \frac{\cancel{400}^{\cancel{40}^{20}} \times \sqrt{3}}{{}_1\cancel{10} \times \cancel{2}_1}$$

$$= 20\sqrt{3} m$$

(d) $T_f = 2s + 2s$

$$= 4s.$$

Q) Find the ∠ of projectile for a body to have same horizontal range for max. height.

Sol) $R = H$

$$\frac{\cancel{u^2} \sin 2\theta}{\cancel{g}} = \frac{\cancel{u^2} \sin^2\theta}{2\cancel{g}}$$

$$2\cancel{\sin\theta}\cos\theta = \frac{\sin^{\cancel{2}}\theta}{2}$$

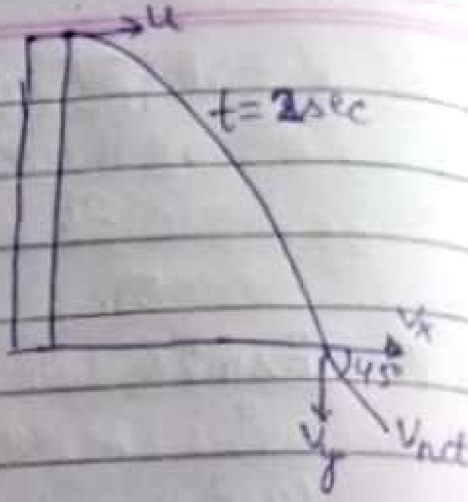
$$4 = \tan\theta$$

$$\theta = \tan^{-1}(4)$$

Q) A body thrown horizontally from the top of tower & strikes the ground after 2 sec. with an angle of 45° with the horizontal. Find the height of tower & speed with which the body projected. ($g = 9.8 m/s^2$)

Ans $t_d = 2 sec \Rightarrow t_f = 4 sec$

$\theta = 45°$



$\cancel{s = ut + \frac{1}{2}at^2}$

$\cancel{s = 0 + \frac{1}{2} \times 9.8 \times 2^2}$

$\cancel{\boxed{s = 19.6m}}$

$S_y = u_y t + \frac{1}{2}a_y t^2$

$S_y = 0(t) + \frac{1}{2}(9.8)(3)^2$

$S_y = \frac{9.8}{2} \times 3$

$\boxed{S_y = 44.1m}$

$\tan 45° = \frac{v_y}{v_x} = 1 \quad -(i)$

$v_y = u_y + a_y t$

$= 0 + (9.8)(3)$

$v_y = 29.4 m/s$

Putting $v_y = 29.4 m/s$ in (i)

$\frac{29.4}{v_x} = 1$

$\boxed{v_x = 29.4 m/s}$

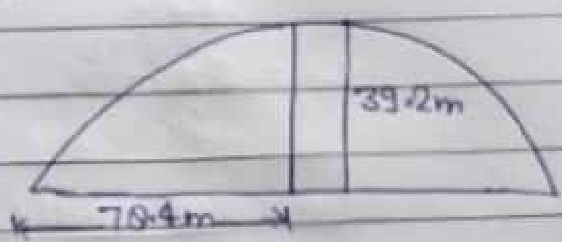
$v_x = u_x + a_x t$

$29.5 = U_x + (0)t$

$\boxed{U_x = 29.5 m/s}$

Q) A body stands at 78.4 m from a building & throws a ball which just enters a window 39.2 m above the ground. Calculate the velocity of projection.

Ans $H = \frac{u^2 \sin^2\theta}{2g} = 39.2$ — (i)



$R = \frac{u^2 \sin 2\theta}{g} = 2 \times 78.4\,m$ — (ii)

Dividing ① by ②

$$\frac{\cancel{u^2}\sin^2\theta}{2\cancel{g}} \times \frac{\cancel{g}}{\cancel{u^2}\sin 2\theta} = \frac{\cancel{39.2}}{2 \times \cancel{78.4}}$$

$$\frac{\sin^2\theta}{4\sin\theta\cos\theta} = \frac{1}{4}$$

$$\frac{\sin\theta}{4\cos\theta} = \frac{1}{4}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{4}$$

$$\tan\theta = 1$$

$$\theta = 45^\circ$$

So, $R = \frac{u^2 \sin 2\theta}{g}$

$$78.4 \times 2 = \frac{u^2 \sin 2 \times 45^\circ}{9.8}$$

$$2 \times 78.4 = \frac{u^2 \times 1}{9.8}$$

$$\sqrt{78.4 \times 2 \times 9.8} = u^2$$

$$\boxed{u = 39.2\,m/s}$$

Q) The blades of an aeroplane propeller are rotating at 600 revolution per minute (rpm).

Ans $\omega = \frac{2\pi N}{60}$

$\omega = \frac{2\pi \times \cancel{600}^{10}}{\cancel{60}_1}$

$\boxed{\omega = 20\pi \text{ rad/s}}$

Q) A particle is projected with velocity u so that its horizontal range is thrice of greatest height. What is range.

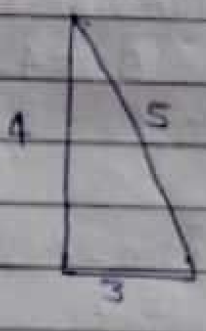
Ans $R = 3H$

$\frac{\cancel{u^2}\sin 2\theta}{\cancel{g}} = \frac{3\cancel{u^2}\sin^2\theta}{2\cancel{g}}$

$2\sin\theta\cos\theta = \frac{3\sin^2\theta}{2}$

$\frac{4}{3} = \frac{\sin\theta}{\cos\theta}$

$\frac{4}{3} = \tan\theta$



$\Rightarrow \sin\theta = \frac{4}{5}$ & $\cos\theta = \frac{3}{5}$

Now,

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$

$R = \frac{u^2}{\,} \times \frac{2\sin\theta\cos\theta}{g}$

$$= \frac{u^2}{g} \times 2 \times \frac{4}{5} \times \frac{3}{5}$$

$$= \frac{24u^2}{25} \ m$$

Q) A body of mass m is thrown with a velocity v at an angle of 30° to the horizontal & another body B at angle of 60° with same velocity $\frac{R_A}{R_B}$ & $\frac{H_A}{H_B}$.

Sol) $\frac{R_A}{R_B} = \frac{\frac{\cancel{v^2}\sin 2\theta_1}{\cancel{g}}}{\frac{\cancel{v^2}\sin 2\theta_2}{\cancel{g}}}$

$= \frac{\sin 2\theta_1}{\sin 2\theta_2}$

$= \frac{\cancel{2}\sin\theta_1\cos\theta_1}{\cancel{2}\sin\theta_2\cos\theta_2}$

$= \frac{\sin 30° \cos 30°}{\sin 60° \cos 60°}$

$= \frac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} \times \frac{1}{2}}$

$\boxed{R_A : R_B = 1:1}$

$\frac{H_A}{H_B} = \frac{\cancel{v^2}\sin^2\theta_1}{\cancel{v^2}\sin^2\theta_2}$

$= \frac{\sin^2 30°}{\sin^2 60°}$

$= \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{\frac{1}{4}}{\frac{3}{4}}$

$= \frac{1}{3}$

$\boxed{H_A : A_B = 1:3}$

Q) $\vec{F} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k}$

Find torque.

Q) If $u = 28 m/s$ & $\theta = 30°$.
Find - (a) Max. hieght
(b) The time taken by the ball to return to the same level.
(c) Horizontal distance.

Ans (a) $H = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{28^2 \times 1}{2\times 9.8 \times 4}$

$= \frac{784}{78.4}$

$= 10m$

(b) $T_f = \frac{2u\sin\theta}{g}$

$= \frac{\cancel{2}\times 28 \times 1}{9.8 \times \cancel{2}}$

$= 2.9s$

(c) $R = \frac{u^2 \sin 2\theta}{g}$

$= \frac{28^2 . \sqrt{3}}{9.8 \times 2}$

$= \frac{784\sqrt{3}}{19.6}$

$= 40\sqrt{3}$

$= 69.3m$

Q) The x & y coordinate of the particle is given as $x = 5t - 2t^2$

$y = 10t$

Find acceleration at $t = 2\,sec$.

Ans- $x = 5t - 2t^2$

~~$y = 10t$~~

$V_x = \frac{dx}{dt}$

$V_x = 5 \times 1 - 2 \times 2t$

$= 5 - 4t$

Now, $a_x = \frac{dV_x}{dt}$

$= -4\,m/s^2$

$y = 10t$

$V_y = \frac{dy}{dt}$

$V_y = 10$

Now,

$a_y = \frac{dV_y}{dt}$

$= 0$

$\therefore a_R = -4\,m/s^2$.

Q) $\vec{r} = 3\hat{i} - 4\hat{j} + \hat{k}$

$\vec{\omega} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

Find velocity.

Ans $\vec{v} = \vec{\omega} \times \vec{r}$

$$= \begin{vmatrix} i & j & k \\ 5 & -6 & 6 \\ 3 & -4 & 1 \end{vmatrix}$$

$= \hat{i}(-6+24) + \hat{j}(18-5) + \hat{k}(-20+18)$

$= 18\hat{i} + 13\hat{j} - 2\hat{k}$

$R = ut$

$\cancel{y} = \frac{1}{2} g t^2$

Equation of Trajectory (The Parabolic Path)

$$\boxed{y = \frac{1}{2}\frac{gx^2}{u^2}}$$

Velocity after time t

$v^2 = u^2 + 2as$

$v = \sqrt{u^2 + 2as}$

$v = \sqrt{u^2 + \cancel{2} \times g \times \frac{1}{\cancel{2}} g t^2}$

$$\boxed{v = \sqrt{u^2 + g^2 t^2}}$$

$R = u \times T$

$$\boxed{R = u\sqrt{\frac{2h}{g}}} \qquad \left[\because T = \sqrt{\frac{2h}{g}}\right]$$

Ans- Torque $= \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-2-9) + \hat{j}(6-2) + \hat{k}(3+4)$$

$$= -11\hat{i} + 4\hat{j} + 7\hat{k}$$

Q) The frequency ν of vibration of a stretched string depends on its length, mass per unit length & tension T. Derive the formula.

Ans Frequency $(\nu) \propto L^a F^b M^c$

where, L = Length

F = Tension (Force)

M = Mass/unit length

Now,

$$\nu = k L^a F^b M^c$$

$$[T^{-1}] = k[L^a][MLT^{-2}]^b[ML^{-1}]^c$$

$$[M^0 L^0 T^{-1}] = k[L^{a+b-c}][M^{b+c}][T^{-2b}]$$

Equating the dimensions of both sides,

$b + c = 0$ — (i)

$a + b - c = 0$ — (ii)

$-2b = -1$ — (iii)

From (iii), we get

$b = 1/2$

Putting $b = 1/2$ in (i), we get

$c = -1/2$

Putting $b = 1/2$ & $c = -1/2$ in (ii) we get

$a = -1$

$\therefore$ Frequency $(\nu) = K L^{-1} F^{1/2} M^{-1/2}$

$$= \frac{K}{L}\sqrt{\frac{F}{M}}$$

Q) $y = a \sin(\omega t - kx)$, where t = time & x = distance. Find dimension of ω & k.

Soln $\theta = M^0 L^0 T^0 = 1$

So,

$\omega t - kx = M^0 L^0 T^0$

Now,

Dimension of $\omega = \frac{M^0 L^0 T^0}{T} = [T^{-1}]$

Dimension of $k = \frac{M^0 L^0 T^0}{L} = [L^{-1}]$

Q) Find the dimension of $a \times b$ in the relation $P = \frac{b - x^2}{at}$

Ans→ According to principle of homogeneity of equation.

$x^2 = b$

$b = [L^2]$

& $P.at = b - x^2$

$[ML^2T^{-3}].a.[T] = [L^2]$

$$a = \frac{[L^2]}{[ML^2T^{-2}]}$$

$$= [M^{-1}T^2]$$

Q) Find dimension of Young's Modulus of Elasticity.

Ans Young's Modulus $(Y) = \frac{\text{Stress}}{\text{Strain}}$ —(i)

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Now, stress = $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

& Strain = $\frac{\text{Change in Length}}{\text{Original Length}} = \frac{\Delta L}{L} = \frac{[M^0L^0T^0]}{[M^0L^0T^0]} = 1$

Putting dimension of stress & strain in(i)

$Y = [ML^{-1}T^{-2}]$

Hook's Law:

Stress ∝ Strain

Stress = k strain

$k = \frac{\text{Stress}}{\text{Strain}}$

$E = \frac{\text{Stress}}{\text{Strain}}$ [where E = k] — (i)

Now,

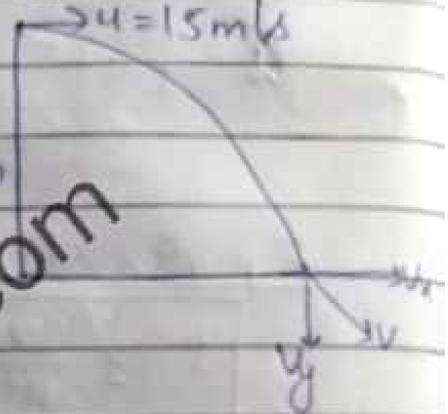
Stress = $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

& strain = $\frac{\text{Change in Length}}{\text{Original Length}} = \frac{\Delta L}{L} = \frac{[L]}{[L]} = 1$

Putting dimensions of stress & strain in(i), we get

$E = [ML^{-1}T^{-2}]$

Q) A hiker stands on the edge of a cliff 490m above the ground & throws this stone horizontally with an initial speed of 15m/s. Neglecting the air resistance, find the time taken by the stone to reach the ground & the speed with which it hits the ground. ($g = 9.8 m/s^2$)



Ans- $T = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2 \times 490}{9.8}}$

$= \sqrt{100}$

$= 10s$

~~$v = u + at$~~

~~$v = 15 + (9.8)(10)$~~

~~$= 15 + 98$~~

~~$= 113m$~~

$v_y = u_y + gt$

$= 0 + 9.8(10)$

$= 98 m/s$

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{15^2 + 98^2}$

$= 99.1 m/s$

Q) The velocity time-graph of an electron starting from rest is given by $v = kt$ where $k = 2m/s^2$. Calculate the distance travelled in 3sec.

Soln ~~$u = 0 m/s$~~

~~Now,~~

~~$a = \frac{dv}{dt}$~~

~~$v = kt,\ k = 2m/s^2$~~

~~Then, $v = 2t$~~

~~$= 2 \times 3 = 6m/s$~~

~~$v = u + at$~~

~~$V = (2)(3) = 6 m/s$~~

$V = kt$

$\frac{dx}{dt} = kt \Rightarrow dx = kt\,dt$

$\int_0^x dx = \int_0^3 kt\,dt$

$x = k\left(\frac{t^2}{2}\right)_0^3$

$x = k\frac{9}{2}$

$x = 2 \times \frac{9}{2}$

$\boxed{x = 9m}$

Q) The acceleration experienced by a boat after the engine is cut off is given by $dv/dt = -kv^3$, where k is constant. If v_0 is the velocity at cut off. Find the magnitude of velocity at time t after the cut off.

Ans- $\frac{dv}{dt} = -kv^3$

$dv = -kv^3 dt$

$\frac{dv}{v^3} = -k\,dt$

$\int_{v_0}^{v} v^{-3} dv = -\int_0^t k\,dt$

$\left[\frac{v^{-3+1}}{-3+1}\right]_{v_0}^{v} = -k[t]_0^t$ $\quad\left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$

$\left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -k[t]_0^t$

$\cancel{-}\frac{1}{2}\left[\frac{1}{v^2}\right]_{v_0}^{v} = \cancel{-}k[t]_0^t$

$$\left[\frac{1}{v^2}-\frac{1}{v_0^2}\right] = 2kt$$

$$\frac{1}{v^2}-\frac{1}{v_0^2} = 2kt$$

$$\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt$$

$$\frac{1}{v^2} = \frac{1+2kt v_0^2}{v_0^2}$$

Q) $\vec{A} = -3\hat{j}+4\hat{i}+2\hat{k} \Rightarrow 4\hat{i}-3\hat{j}+2\hat{k}$

$\vec{B} = 2\hat{i}-3\hat{j}+5\hat{k}$

Find (i) $\vec{A}\cdot\vec{B}$

(ii) $\vec{A}\times\vec{B}$

(iii) Angle b/w $\vec{A}$ & $\vec{B}$

Ans(i)- $\vec{A}\cdot\vec{B} = 8+9+10$

$= 27$

(ii)- $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 2 \\ 2 & -3 & 5 \end{vmatrix}$

$= \hat{i}(-15+6)+\hat{j}(4-20)+\hat{k}(-12+6)$

$= -9\hat{i}-16\hat{j}-6\hat{k}$

(iii)- $\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

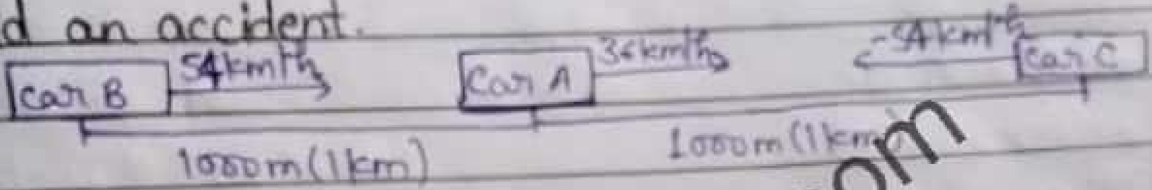
$27 = \sqrt{29}.\sqrt{38}\cos\theta$

$\cos\theta = \frac{27}{\sqrt{29}.\sqrt{38}}$

$\theta = \cos^{-1}\left(\frac{27}{\sqrt{29}.\sqrt{38}}\right)$

Q) On two lane road, car A is travelling with a speed of 36km/h. Two cars B&C approach car A in opp. direction with a speed of 54km/h each. At a certain instant, when the distance AB is equal to AC both being 1km, B decides to overtake A before C does. What min. acceleration of car B is required to avoid an accident.

Ans→



$V_A = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$

$V_B = 54 \text{ km/h} = 54 \times \frac{5}{18} = 15 \text{ m/s}$

$V_C = -54 \text{ km/h} = -54 \times \frac{5}{18} = -15 \text{ m/s}$

I case:

Relative velocity of C w.r.t to A $= V_{CA} = V_C - V_A$

$= -15 - 10$

$= -25 \text{ m/s}$

Time that C requires to just cross A

$T = \frac{\text{Distance}}{\text{Speed}} = \frac{1000}{-25} = 40 s$

II case:

Relative velocity of B w.r.t to A $= V_{BA} = V_B - V_A$

$= 15 \text{ m/s} - 10 \text{ m/s}$

$= 5 \text{ m/s}$

$s = 1 \text{ km} = 1000 \text{ m}$

$t = 40 \text{ sec}$

Now,

$s = ut + \frac{1}{2}at^2$

$1000 = 5(40) + \frac{1}{2}a(40)^2$

$1000 = 200 + \frac{1}{2}a(1600)$

$800 = 800a$

$\boxed{a = 1 m/s^2}$

Thus, $1 m/s^2$ is the min. acceleration that B requires to avoid accident.

Q) A car moving with a speed of 50 km/h can be stopped by brakes after at least 6m. What will be the min. stopping distance if the same car is moving at a speed of 100 km/h.

Ans- I case:

$u = 50 km/h = 50 \times \frac{5}{18} = \frac{125}{9} m/s$

$v = 0 m/s$

$s = 6m$

Now,

$v^2 = u^2 + 2as$

$0 = \left(\frac{125}{9}\right)^2 + 2(a)(6)$

$-12a = \frac{15625}{81}$

$a = \frac{-15625}{972} m/s^2$

$a = -16 m/s^2$

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II case:

$u = 100\,km/h = 100 \times \frac{5}{18} = \frac{250}{9}\,m/s$

$v = 0\,m/s$

$a = -16\,m/s^2$

Now,

$v^2 = u^2 + 2as$

$0 = \left(\frac{250}{9}\right)^2 + 2(-16)(s)$

$32s = \frac{62500}{81}$

$s = \frac{\cancel{62500}^{\cancel{31250}\ 15625}}{81 \times \cancel{32}\,\cancel{16}\,8}$

$\boxed{s = 24.11\,m}$