Laws of Motion :-

⇒ Inertia :- Inertia is an inherent property of a body by virtue of which it cannot change its state (i.e. rest or uniform motion) by itself".
It says that every body in the universe does have a property which is hidden in itself and because of this property the body is unable to change its state by itself, i.e. from state of rest to state of motion or vice-versa or even its direction. This inertia can be categorized into three types, namely:
(a) Inertia of Rest
(b) Inertia of motion
(c) Inertia of direction

Inertia of Rest :- Inertia of rest is the inability of a body by virtue of which it can't change its state of rest to state of motion. That means

any body which is at rest continues to be in the state of rest only and it can't go further into state of motion by itself.

Eg:- (1) Passengers standing or sitting loosely in a bus experience jerk in the backward direction when the bus suddenly starts moving. This is due to the fact that when the bus suddenly starts its motion, the lower parts of the human body shares the motion but the upper part tends to remain at rest due to inertia of rest.

(2) When a bullet is fired into a tightly-fitted glass pane from a reasonably close range, it makes a clear circular hole in the glass pane. This is due to the fact that particles of glass around the hole tend to remain at rest due to inertia of rest. So, they are unable to share the fast motion of the bullet.

⇒ **Inertia of Motion:-** Inertia of motion is the inability of a body by virtue of which it cannot change its state of uniform motion along a straight line to state of rest. That means any body which is in uniform motion can't come to rest by itself until and unless some external force acts on it.

Eg:- An athlete runs for some distance before taking a long jump. In this way, the athlete gains momentum and this inertia of motion helps him in taking longer jump.

⇒ **Inertia of Direction:-** Inertia of direction is inability of a body by virtue of which it can't change its direction by itself. This means a body moving along a straight line can't change its direction by itself, until and unless it is acted upon by any external force.

Eg:- (1) When a running car suddenly takes a turn, the passengers experience a jerk in the outward direction. This is because the passengers tend to maintain their original direction of motion due to inertia of direction.

(2) A stone tied to one end of a string is whirled in a horizontal circle. When string breaks, the stone tends to fly off tangentially along a straight line. This is due to inertia of direction.

Note:- The mass of the body is the indirect measure of the inertia of that body.

"Force is that which pushes or pulls the body or tends to change the state of rest or of uniform motion in a straight line".

(a) It produces or tries to produce motion in a body at rest.

(b) It stops or tries to stop a moving body.

(c) It changes or tries to change the direction of motion of body.

(d) It produces a change in the shape of the body.

Classification of forces :-

There are different types of forces in our Universe. Based on the nature of the interaction between two bodies, forces may be broadly classified as under:-

Force

- System wise
  - External
  - Internal
- Source (origin)
  - Field
  - Contact
- Reference Frame
  - Real
  - Pseudo
- Nature (Workdone)
  - Conservative
  - Non-Conservative

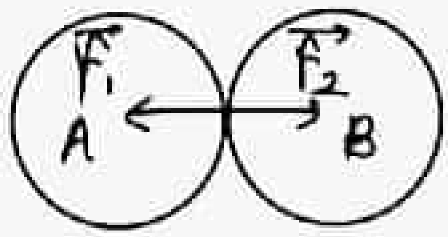
(a) Contact force:- The force exerted by one surface over the surface of another body when they are physically in contact with each other is known as Contact force.

If two surfaces that are coming into contact are perfectly smooth, then the entire contact force will act only normal to their surface of contact and it is known as "Normal force".

If two surfaces that are coming into contact are rough surfaces, then one component of this contact force acts perpendicular to their surface of contact and the other component of this force acts in tangential direction to their surface of contact and this component is known as "force of friction".

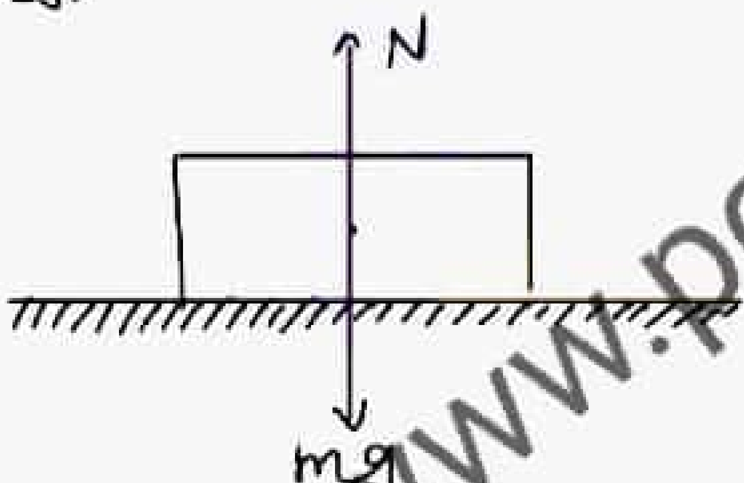
Normal Reaction, Tension, Friction etc are the eg. of various contact forces.

Normal Reaction: The forces $\vec{F_1}, \vec{F_2}$ acting on A and B respectively act away from the surface of contact and prevent the two bodies from "Occupying the same space".



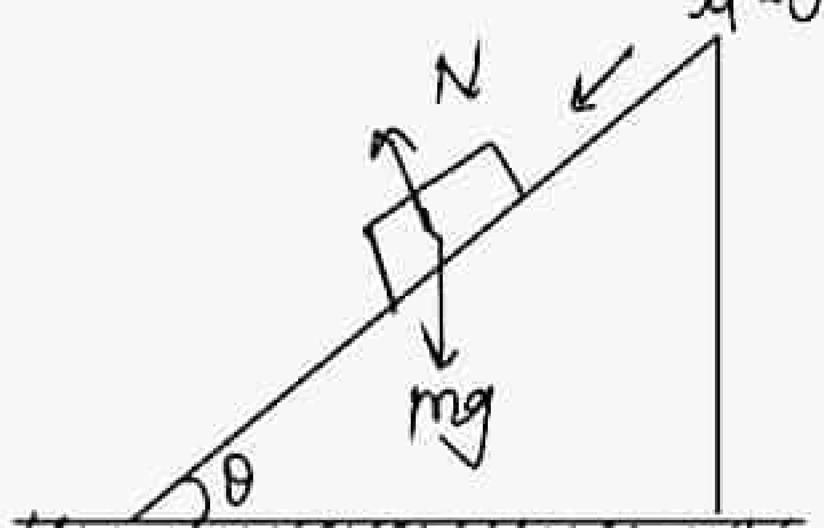
If $\vec{F_1}$ is the action, $\vec{F_2}$ is reaction, they are equal in magnitude but opposite in direction. Further, $\vec{F_1}$ and $\vec{F_2}$ are both perpendicular to the surfaces in contact and note that they act on two different bodies.

Eg:-



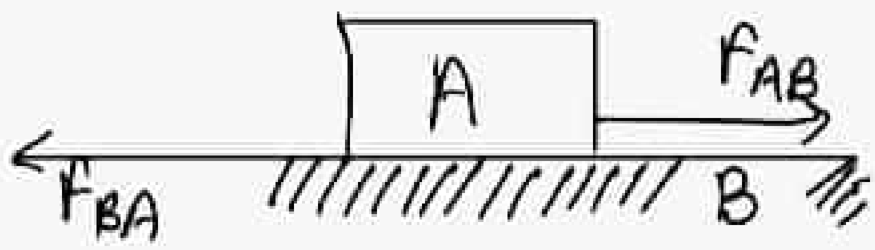
$N = mg$

N is the normal reaction of the ground on the stone



$N = mg \cos\theta$

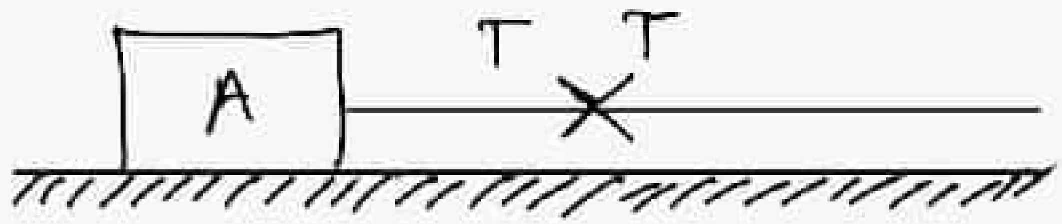
⇒ **Friction:-** It is a force that acts between bodies in contact with each other along the surface of contact and it opposes relative motion between the two bodies. The direction of frictional force on A is opposite to that of direction of frictional force on surface B and magnitude is same for both.



⇒ **Tension (T):-** When a string, thread or wire is held taut, the ends of the string or thread (or wire) pull on whatever bodies are attached to them in the direction of the string. This force is known as Tension.

If the string is massless then the tension T has the same magnitude at all points throughout the string.

Eg:- (i) Tension in a string:- for a block A pulled by a string.



⇒ Spring forces:- Whenever a spring is compressed or extended, the elastic force developed in the spring which helps the spring to restore its original position is known as spring force.

In an extended (or compressed) spring, force is proportional to the magnitude of extension (or compression)

$F \propto x$, in magnitude, but opposite in direction,

So, $F = -kx$, where k is a positive constant, also known as the spring constant of the spring.

x is the compression or elongation from the natural length.

Eg:- In case of the spring the tensions are oppositely directed on the block 'm' and on the roof R.

⇒ Non- Contact force :- Bodies can exert forces on each other without actual physical contact. This is known as action at a distance. Such forces are known as non-contact forces (or) field forces.
Eg. Gravitational force, Electrostatic forces etc

For the moment, we deal with actual forces. Suffice it to say that there exist pseudo-forces acting in a non- inertial frame of reference.

Newton's first law:-

Every body continues to be in the state of rest or of uniform motion in a straight line until and unless it is compelled to change the state of the body by an unbalanced force.

For better understanding we can divide this statement into two parts:-

(1) Every body continues to be in its state of rest until and unless some external force compels it to change the state of rest".

This part of the law is self explanatory and self evident as we come across several eg. in our daily life like all inanimate objects will continue to be in the same place where they are put until they are disturbed by some external agents.

(2) Every body continues in its state of uniform motion in a straight line unless external force compels it to change that state".

The second part of the statement can't be readily understood as on the surface of the earth because of various types of frictional or resisting forces. For eg. when a ball is rolled on a horizontal surface the ball will come to halt after some time however smooth the surface may be, as we can't eliminate force of friction completely.

⇒ Momentum (linear) :-

Momentum is defined as the ability of a body by virtue of which it imparts or tends to impart its motion along a straight line.
Mathematically, momentum (p) is measured as the product of mass (m) and velocity (v) of the body.
As velocity is a vector quantity, momentum is also a vector quantity.

$$\vec{p} = m\vec{v} \quad \text{or} \quad p = mv$$

Unit :- It's unit is kg-m/s in SI system and gm-cm/s in CGS system.

Dimensions:- Its dimensions are MLT^{-1}

Eg:- (1) A block of mass 2 kg is moving with a velocity of $2\hat{i} - \hat{j} + 3\hat{k}$ m/s. find the magnitude and direction of momentum of the block with the x-axis.

sol^n:- The magnitude of momentum is $2\sqrt{14}$ kgm/s and the direction is at an angle of $\cos^{-1}\sqrt{2/7}$ with x-axis

(2) When a force of constant magnitude always act perpendicular to the motion of a particle then:-

(a) Velocity is constant (b) Acceleration is constant

(c) KE is constant (d) None of the above

sol^n:- Force will provide centripetal acceleration, so it will move in a circular path therefore K.E. is constant because speed remains unchanged.

Newton's Second law of Motion :-

We have already studied the Newton's 1^{st} law which has given us a qualitative idea about force. Now, we will study about Newton's II^{nd} law which gives us a quantitative idea about force. Whenever a cricketer catches a ball he allows a longer time for his hands to stop the ball. Otherwise the ball will hurt the cricketer. If you observe this incident carefully you can easily understand that cricketer is applying some force on ball in order to make the momentum of the body zero. And also we can understand that the magnitude of the retarding force that cricketer applies on the ball in order to stop depends on two factors.

(1) The momentum of the ball and

(2) Time for which he is applying the force

These type of observations lead Newton to state his second law of motion.

The rate of change of momentum of a body is directly proportional to the applied force and the change take place in the direction of the force.

So for a body with constant mass,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \quad \text{or,} \quad \frac{d}{dt}(m\vec{v}) \propto \vec{F}$$

$$\text{or,} \quad m\frac{d\vec{v}}{dt} \propto \vec{F} \quad ; \quad \vec{F} = Km\left(\frac{d\vec{v}}{dt}\right)$$

where K is a constant. With proper choice of units, K = 1. Thus,

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

Unit of force:- Its unit is Newton in SI System and dyne in CGS system.

Dimensions:- $[MLT^{-2}]$

Note:- (1) The 2^{nd} law of motion is a vector law. There are actually three equations, one for each component of the vectors:-

$$f_x = \frac{dp_x}{dt} = ma_x ; \quad f_y = \frac{dp_y}{dt} = ma_y ; \quad f_z = \frac{dp_z}{dt} = ma_z$$

Note (2):- The 2^{nd} law of motion is strictly applicable to a single point particle. The force F here stands for net external force on the particle.

Eg:- A body of mass $m = 1$ kg falls from a height $h = 20$m from the ground level

(a) What is the magnitude of total change in momentum of the body before it strikes the ground?

(b) What is the corresponding average force experienced by it? ($g = 10 m/sec^2$)

Solⁿ:- (a) Since the body falls from rest ($u = 0$) through a distance h before striking the ground, the speed v of the body is given by kinematics eqⁿ.

$v^2 = u^2 + 2as$; Putting $a = g$ and $s = h$

We obtain $v = \sqrt{2gh}$

$\Rightarrow$ The magnitude of total change in momentum of the body

$\Rightarrow \Delta p = |mv - 0| = mv$, where $v = \sqrt{2gh}$

$\Rightarrow \Delta p = m\sqrt{2gh} \Rightarrow \Delta p = (1)\sqrt{(2 \times 10 \times 20)}$ kgm/sec

$\Rightarrow \Delta p = 20$ kgm/sec

(b) The average force experienced by the body =

$$f_{av} = \frac{\Delta p}{\Delta t}$$

where Δt = time of motion of the body = t (say). We know $\Delta p = 20$ kgm/sec. Therefore we will have to find t using the given data. We know from kinematics that,

$$s = ut + \frac{1}{2}at^2 \Rightarrow h = \frac{1}{2}gt^2 \quad (u = 0)$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} \quad \therefore f_{av} = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{t}$$

Putting the general values of Δp and t we obtain

$$F_{av} = \frac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg = \vec{F}_{av} = m\vec{g}$$

where mg is the wgt (W) of the body and $\vec{g}$ is directed vertically downward. Therefore the body experiences a constant vertically downward force of magnitude mg.

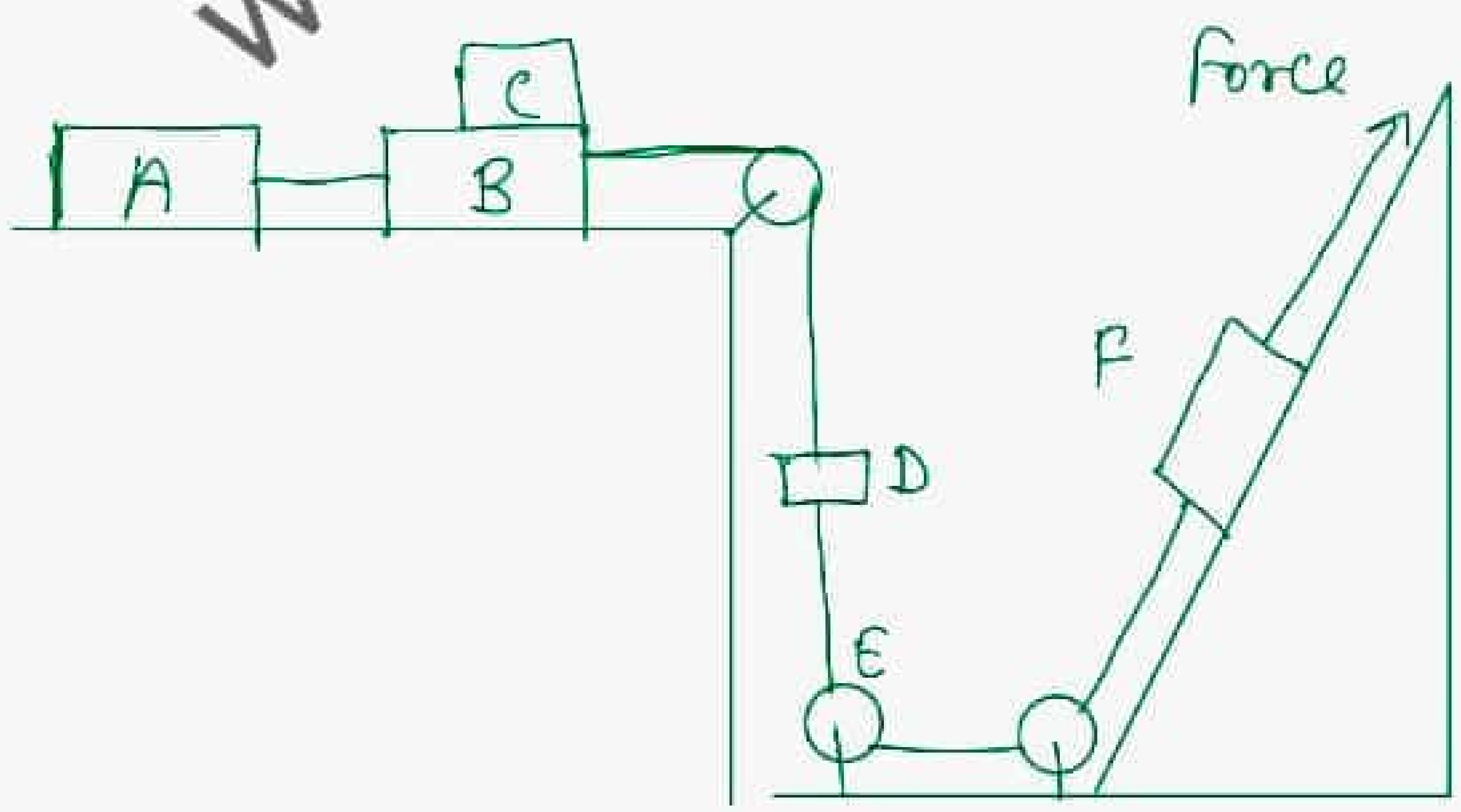
Working with Newton's first and Second law:-

Before trying to write an eqn from Newton's law, we should very clearly understand which particle we are considering. In any particle situation, we deal with extended bodies which are collection of a large no. of particles. The laws stated a force may be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing eqns from Newton's law is as follows:-

1^{st} Step:- Decide the System :-

The 1^{st} step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

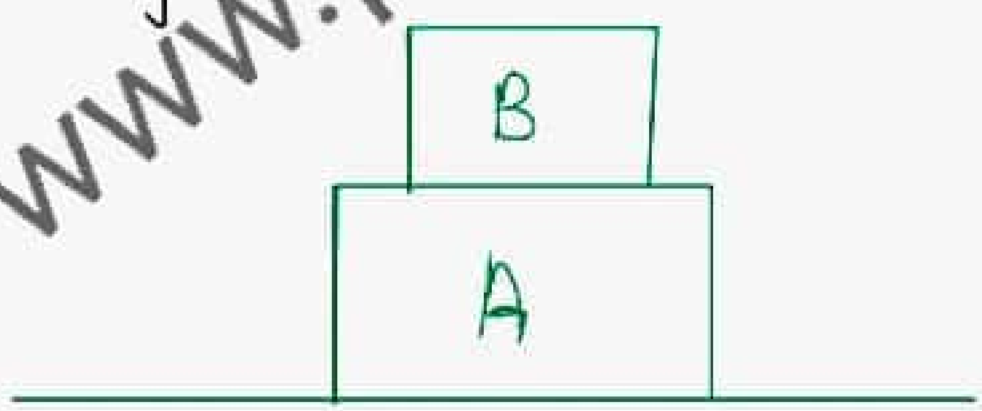
Here the distance covered by all the blocks is same but they all cannot be taken as a system because even though magnitude of acceleration is same but the direction of acceleration in all the blocks is not same.



2nd Step:- Identify the forces:-

Once the system is decided, make a list of forces acting on the system due to all the objects other than system. Any force applied by the system should not be included in the list of the forces.

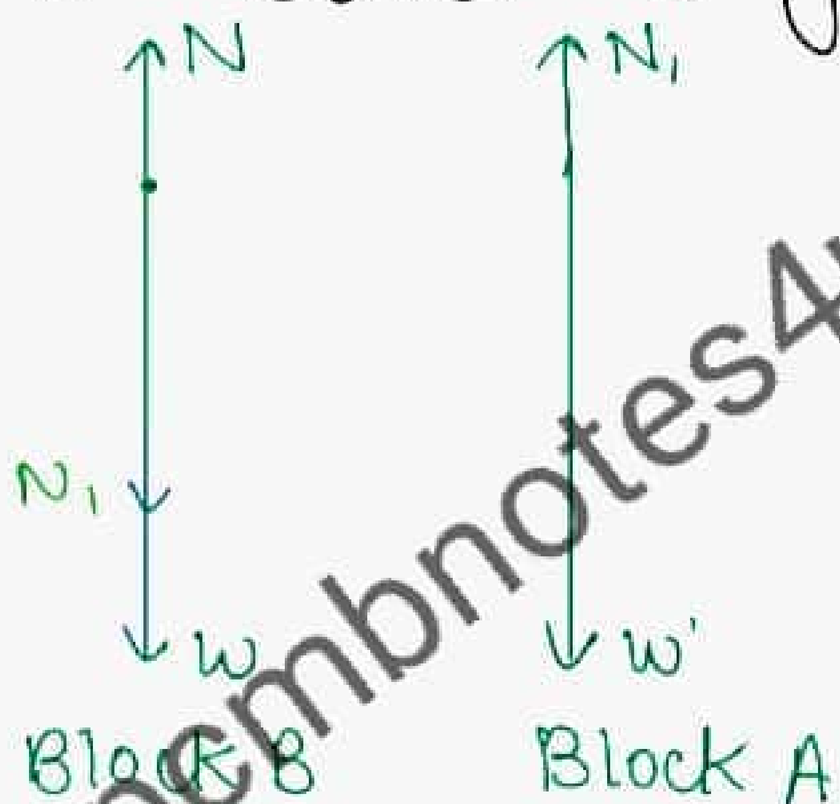
Consider the situation shown in the fig. where small block of mass m is kept on bigger block of mass M. The load presses lower block, the lower pushes the upper block, the bigger block presses the floor downward, the floor pushes the block upward, the earth attracts the block.



3rd Step:- Make a free body diagram:-

Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this

point as the common origin.
The forces may lie along a line, may be distributed in a plane (copolar) or may be distributed in the space (non-planer). Indicate the magnitude and directions of the forces in the diagram. This is called a free body diagram



4th Step:- chose axes and write equations:-
Any three mutually perpendicular directions may be chosen as the x-y-z axes. Some suggestions for choosing the axes to solve problems are:-
(a) If the forces are coplaner, only two axes, say x and y, taken in the plane of forces are needed
(b) Chose the x-axis along the direction in which the system is known to have or is likely to have the

acceleration.

(c) If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes.

(d) Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration. Write the components of the forces along the y-axis and equate the sum to zero.

Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

Impulse:- The change in momentum is called impulse. In many situation's a large force acts for a short time, in this case this force is called Impulsive force

Mathematically it is described as the product of force and time.

$\therefore$ Impulse $(J) = f \cdot t$

$\therefore$ Impulse $(J) = mv - mu$ and since force is variable,

hence $J = \int_{t_1}^{t_2} f\,dt$

The area under f-t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$.

Unit of Impulse:- The unit in S.I. System is kgm/sec or newton-second.

Dimension :- $M^1 L^1 T^{-1}$

Eg:- (1) Automobiles are provided with spring shocker systems. When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, thereby reducing the impulsive force. This minimises the damage to the vehicle.

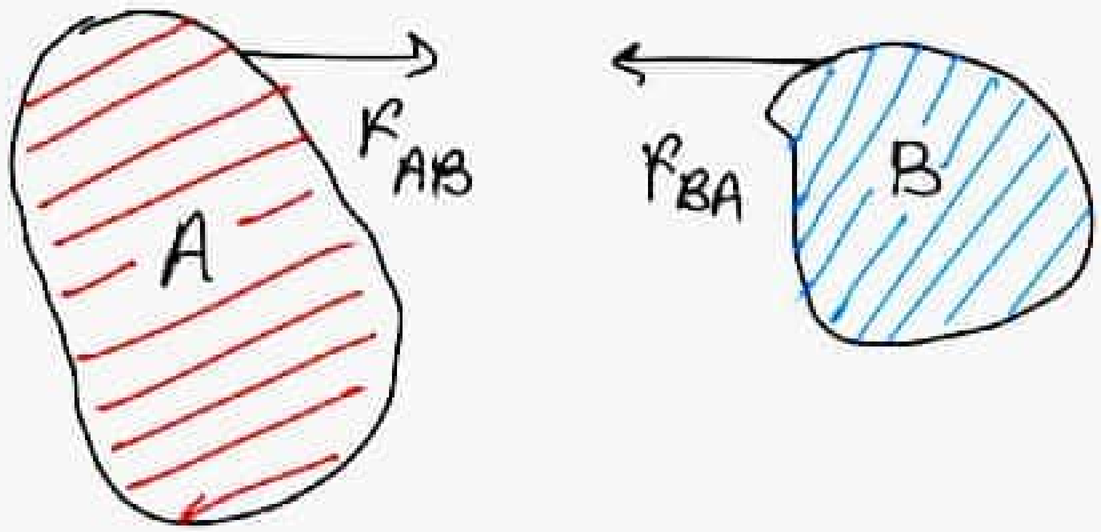
(2) A man falling from a certain height receives more injuries when he falls on a marble floor than when he falls on a heap of sand. This is because the marble floor does not yield under the wgt of the man. The man is stopped abruptly. A finite change of momentum takes place in a very short interval of time. But when he falls on a heap of sand, the sand yields under the wgt of the man and this increases the time interval. So, it reduces the force exerted by sand on man.

(3) It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity. This is because cricket ball will have more momentum than tennis ball due to its heavier mass. The change in momentum in case of cricket ball is more. Hence, more force is required to stop cricket ball than tennis ball.

Newton's third law of Motion:-

Now we have understood the qualitative and quantitative definitions of force from Newton's first and second laws. But how are the forces between two bodies related to each other if at all? The answer is provided by the third law of motion.

Every action has an equal and opposite reaction, which are equal in magnitude and opposite in direction.



Consider two bodies A and B interacting with each other by means of forces.

$\vec{F}_{AB}$: the force exerted by body B on A

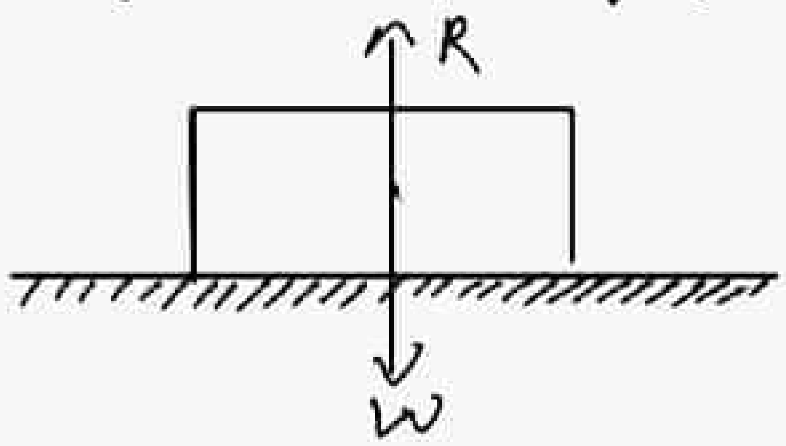
$\vec{F}_{BA}$: The force exerted by the body A on B.

Acc. to Newton's 3rd law. $\vec{F}_{AB} = -\vec{F}_{BA}$ (equal in magnitude and opposite in direction)

That may look fine, but it apparently, raises a lot of questions. for eg., if a horse pulls a cart and cart pulls the horse backward, how does the cart moves forward at all?

If we observe we will find that the forces acting on the horse and the cart, though equal and opposite, they are acting not on the same body, rather, two bodies. It can produce equilibrium neither in horse nor in cart.

Eg:- (1) Consider a body of wgt. W resting on a horizontal surface. The body exerts a force (action) equal to wgt. W on the surface. The surface exerts a reaction R on the body in the upward direction such that W = R or in vector notation, $\vec{W} = -\vec{R}$



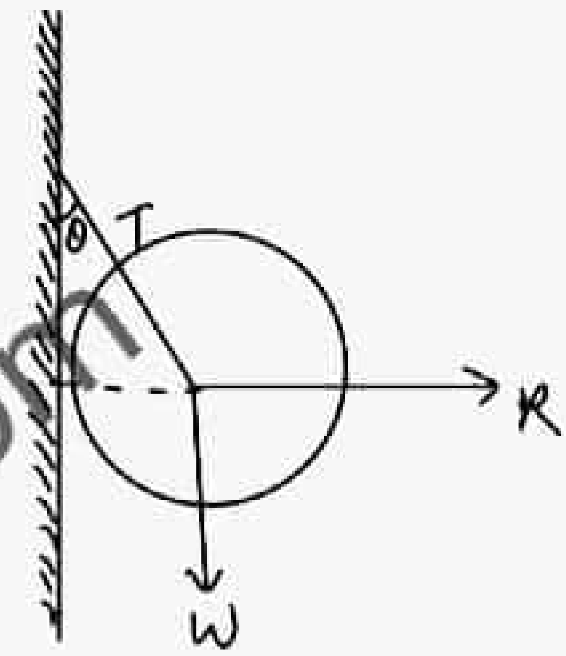
(2) In a lawn sprinkler, when water comes out of the curved nozzles, a backward force is experienced by the sprinkler. Consequently, the sprinkler starts rotating and sprinkles water in all directions.

(3) In order to swim, a man pushes the water backwards with his hands. As a result of the reaction offered by water to the man, the man is pushed forward

Eg:- A metal of sphere is hung by a string fixed to a wall. The force acting on the sphere are shown in fig. which of the following statement is incorrect

(a) $\vec{F} + \vec{T} + \vec{w} = 0$ (B) $T^2 = R^2 + w^2$

(C) $T = R + w$ (D) $R = w \tan\theta$



Solⁿ:- By the free body diagram

$T \cos\theta = w$

$T \sin\theta = R$

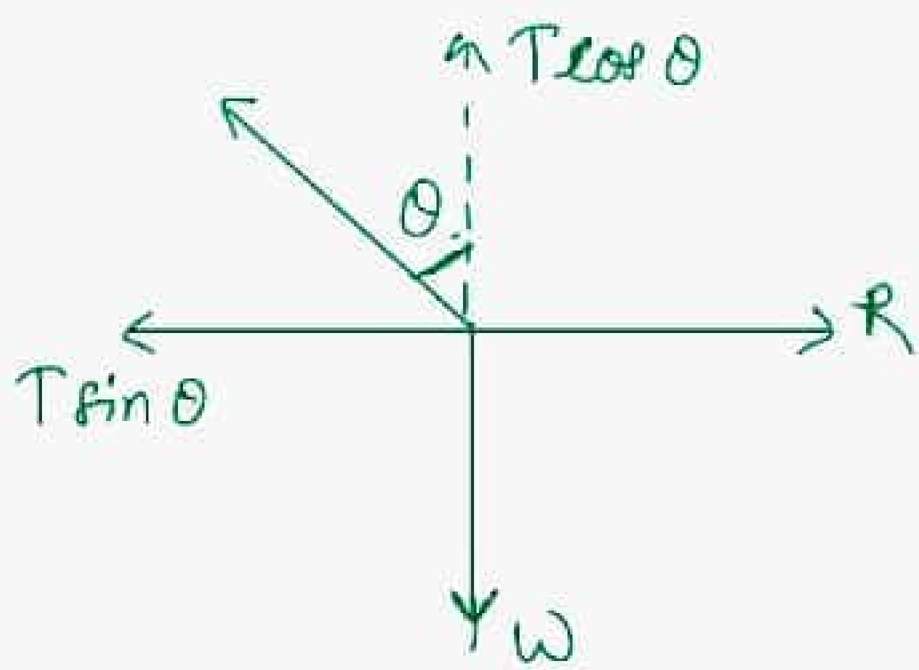
$T^2 = w^2 + R^2$

$\tan\theta = R/w \Rightarrow R = w \tan\theta$

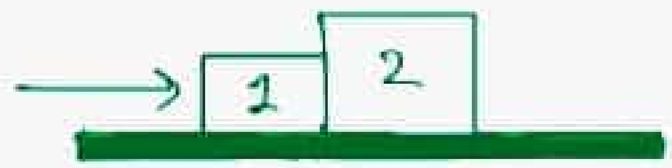
for equilibrium $\Sigma\vec{F} = 0$

$\Rightarrow \vec{F} + \vec{T} + \vec{w} = 0$

Hence (C) is incorrect



Eg:- Two blocks of mass 2 kg and 4 kg are kept in contact with each other on a smooth horizontal surface. A horizontal force of 12 N is applied on the first block due to which they move with certain constant acceleration. Calculate the force between the blocks.



Sol^n:- Let a = Common acceleration of blocks,

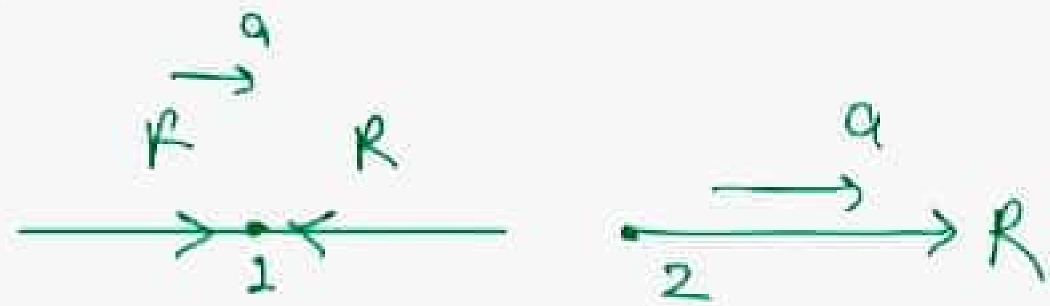
R = force between the two blocks

from FBD of Ist block

$F - R = m_1 a$ ——— (1)

from FBD of block 2,

$R = m_2 a$ ——— (2)



F.B.D.

$\therefore$ from (1) and (2), $F = (m_1 + m_2) \cdot a$

$\therefore a = \dfrac{F}{m_1 + m_2} = \dfrac{12}{6} = 2\,m/sec^2$

$\therefore$ force between blocks $= R = m_2 a = (4 \times 2)\,N = 8\,N$

Frame of Reference:-

It is a conveniently chosen co-ordinate system, which describes the position and motion of a body in space.

⇒ Inertial frame of Reference:-

A Reference frame in which Newton's laws are strictly valid is called as an inertial frame of reference. Any reference frame which is either at rest or in uniform motion with respect to an inertial frame is itself inertial.

⇒ Non - Inertial frame of Reference:-

A reference frame which accelerates or rotates with respect to an inertial reference frame. Newton's law's can be applied in this frame only after applying an additional force (called pseudo force) on the body. Motion of a particle (P) is studied from two frames of references S and S'. S is an inertial frame of reference and S' is a non-inertial frame of reference. At any time, position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}'$ respectively. At the same moment position vector of the origin of S' is $\vec{R}$ with respect to S as shown in the figure.

From the vector triangle OO'P, we get

$$\vec{r}' = \vec{r} - \vec{R}$$

Differentiating this eq^n twice with respect to time we get,

$$\frac{d^2\vec{r}'}{dt^2} = \frac{d^2(\vec{r})}{dt^2} - \frac{d^2}{dt^2}(\vec{R})$$

$$\Rightarrow \vec{a}' = \vec{a} - \vec{A}$$

Here $\vec{a}'$ = acceleration of the particle P relative to S'
$\vec{a}$ = Acceleration of the particle relative to S
$\vec{A}$ = acceleration of S' relative to S.

Multiplying the above eqn by m (mass of the particle) we get,

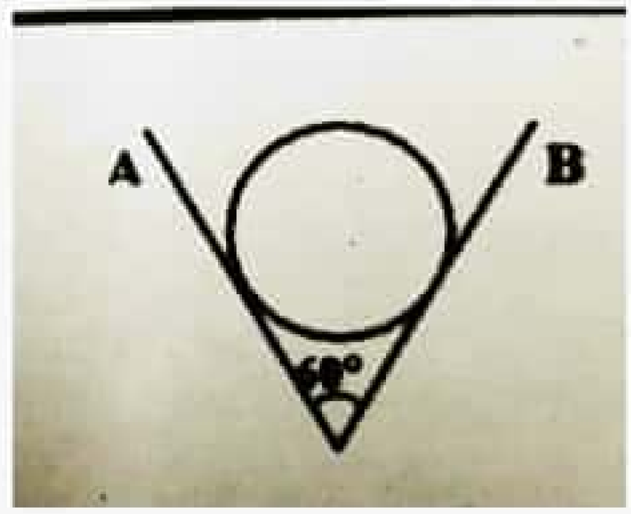
$$m\vec{a}' = m\vec{a} - m\vec{A}$$

$$\Rightarrow \vec{F}' = \vec{F}_{(real)} - m\vec{A}$$

$$\Rightarrow \vec{F}' = \vec{F}_{(real)} + (-m\vec{A})$$

In non-inertial frame of reference an extra force is taken into account in order to apply Newton's laws of motion. The magnitude of this force is equal to the product of the mass of the body and acceleration of the frame and it is always directed opposite to the acceleration of the frame. This force is known as $\boxed{\text{Pseudo force,}}$ because this force does not exist in the inertial frame of reference.

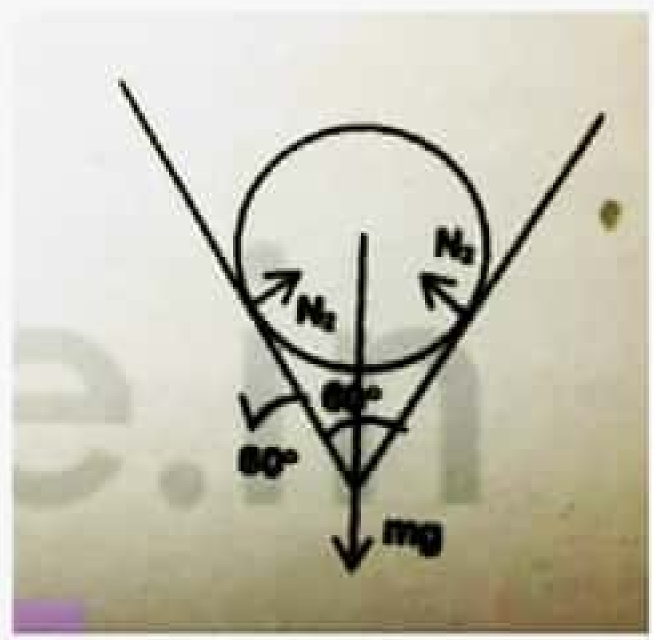
Eg:- An iron sphere weights 10 N and rests in a V shaped through whose sides form a angle 60°. The net force exerted by the walls on the sphere in case as shown in fig.



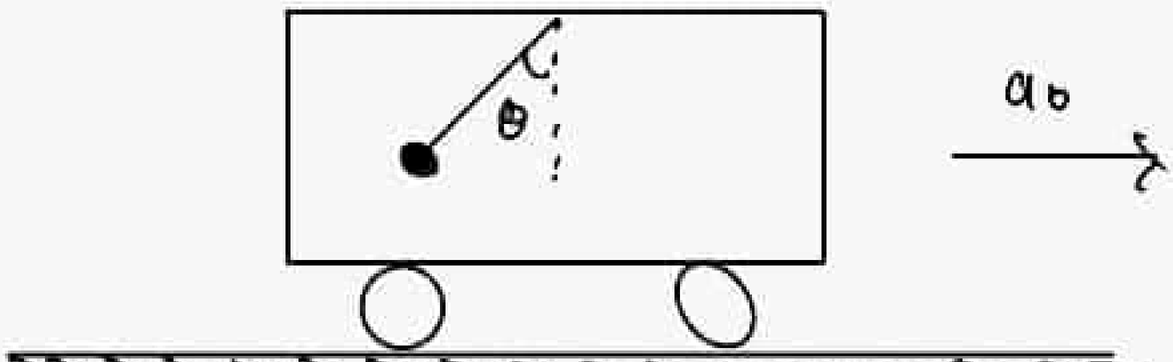
(a) 0 N (b) 20 N

(c) $\frac{10}{\sqrt{3}}$ N (d) 5 N

Sol^n:- By Symmetry $N_1 = N_2$



Eg:- A pendulum of mass m is hanging from the ceiling of a car having an acceleration a_0 with respect to the road in the direction shown. Find the angle made by the string with the vertical.



Sol^n:- Since bob of the pendulum is stationary relative to car

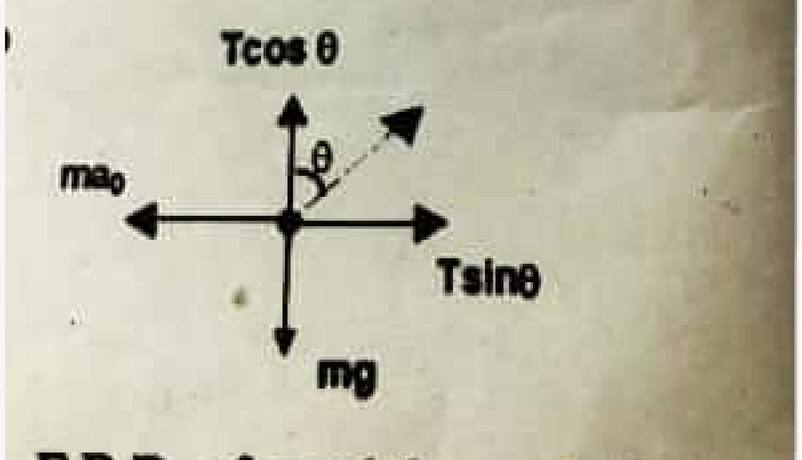
Hence,

$T \sin\theta = ma_0$ (pseudo force) — (1)

$T \cos\theta = mg$ — (2)

Dividing (1) by (2), we get

$\tan\theta = \frac{a_0}{g} \Rightarrow \theta = \tan^{-1}\frac{a_0}{g}$



F.B.D. of pendulum relative to car

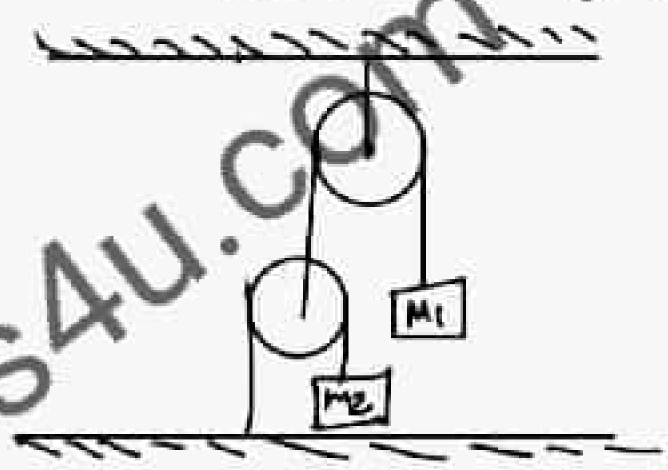
Constraint Relations:-

The eq^{n}s showing the relation of the motions of a system of bodies, in which motion of one body is constrained by motion of other bodies, are called the constraint relations

Applying Newton's laws alone is not sufficient in some cases where the no. of eq^{n}s is less than the no. of unknowns.

In the given diag. for finding the acceleration of the masses there are three unknowns, tensions T_1 acceleration a_1 and a_2 of masses m_1 and m_2.

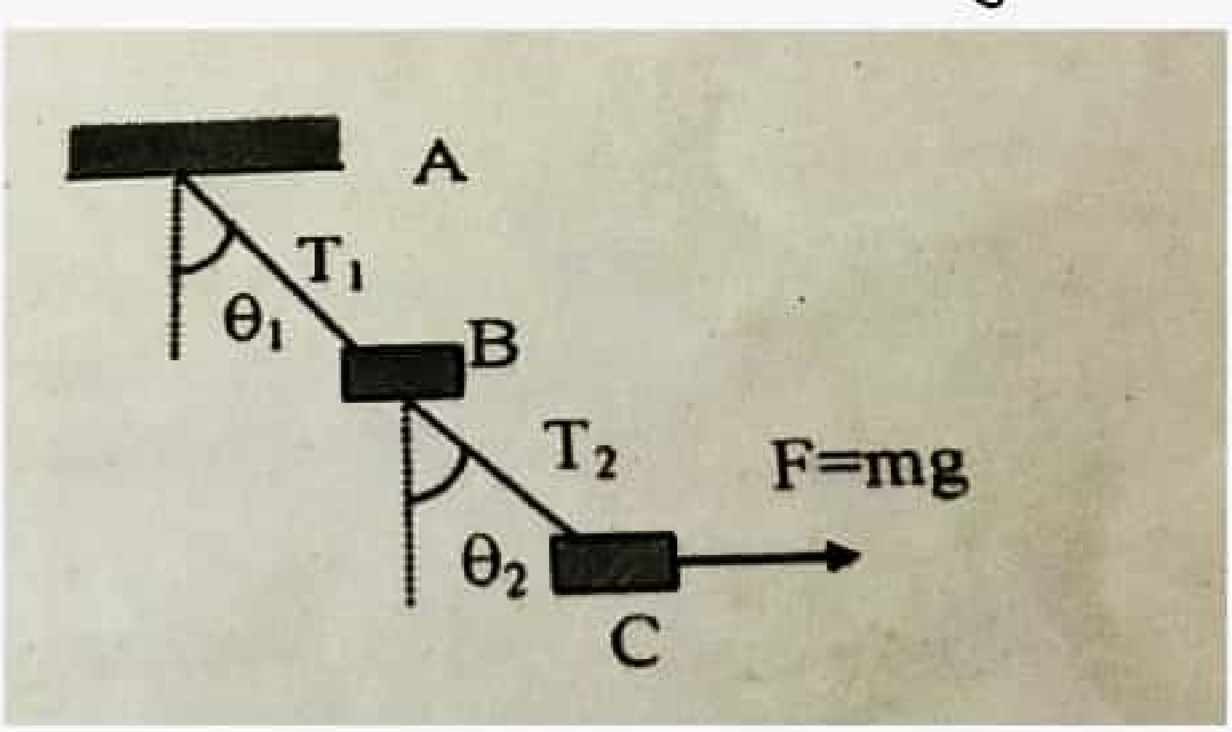
However, we will get only two eqns. Clearly Newton's laws are not sufficient to solve the problem and constraint relations provide additional eqns. When the motions of bodies in a system is constrained because of pulleys, strings, wedges or other factors, we use geometry to develop additional eqns.



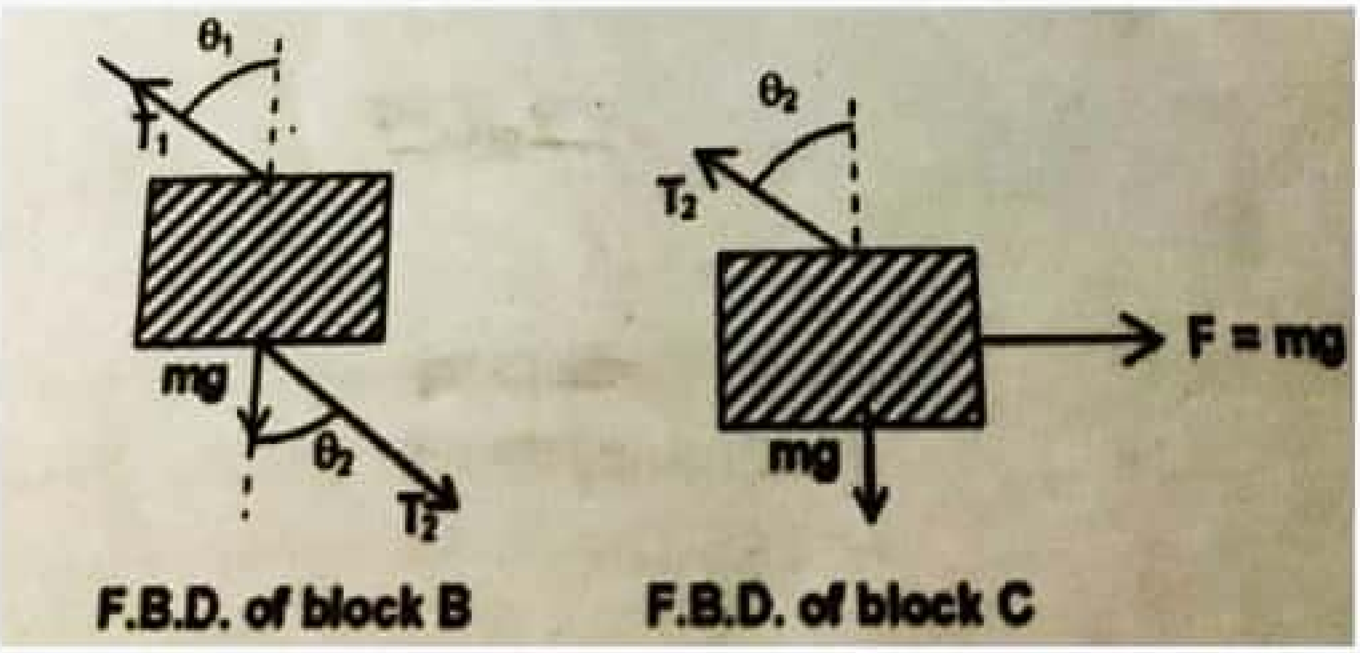
Eg:- The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.

(a) $\tan\theta_1 = 1$ (b) $\tan\theta_2 = \frac{1}{2}$

(c) $T_1 = \sqrt{5}\,mg$ (d) $T_2 = \sqrt{5}\,mg$



Solⁿ:- From F.B.D.



$T_2 \cos\theta_2 = mg$

$T_2 \sin\theta_2 = mg = F$

$\Rightarrow T_2 = \sqrt{2}\, mg$

and $\theta_2 = 45^\circ$

Also $T_1 \sin\theta_1 = T_2 \sin\theta_2$

and $T_2 \cos\theta_2 + mg = T_1 \cos\theta_1$

$T_1 \cos\theta_1 = mg + mg = 2mg$

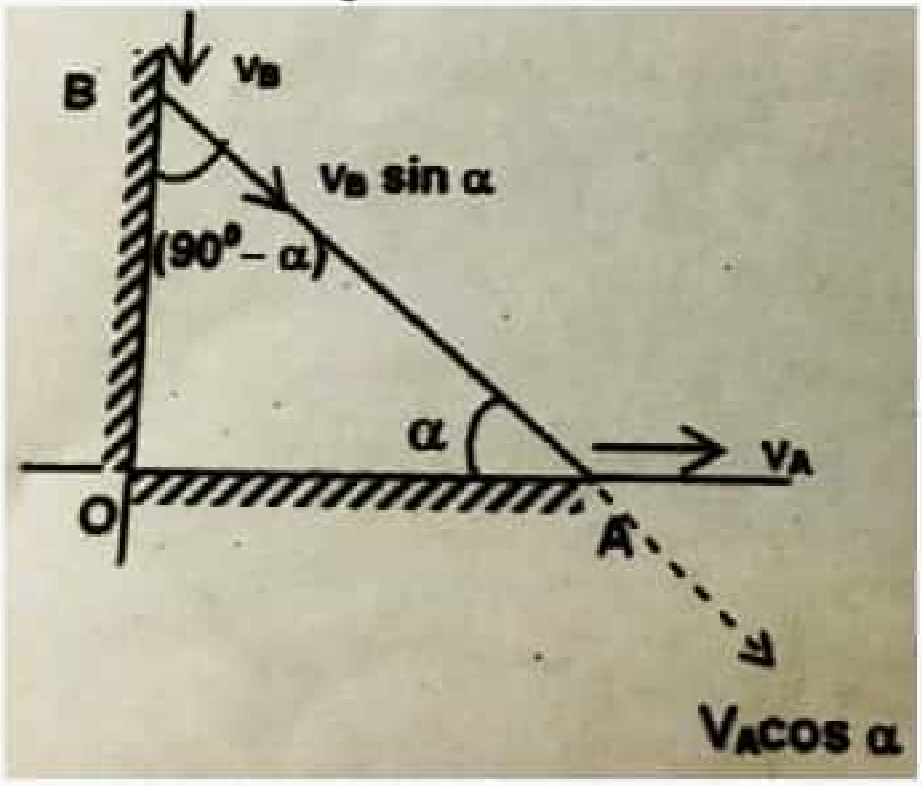
and $T_1 \sin\theta_1 = mg$

Therefore $T_1 = \sqrt{5}\, mg$

So, option (c) is correct

Eg:- A rod is sliding along the wall as shown in fig. Find the ratio of velocity of its ends at the given instant.

Solⁿ→ from the fig.



the length of the rod should be constant, so

$V_A \cos\alpha = V_B \sin\alpha$

$$\frac{V_A}{V_B} = \tan\alpha$$

Application of Newton's laws of motion :- techniques and approach :-

A separate point diag. of the body is drawn showing the different forces exerted by the bodies in the environment, this is known as free body diagram.

Application of Newton's laws to any system (consisting of one or more objects) can be done by following a systematic method. We recommend the following steps in the order given below :-

(1) Draw the complete free body diag. (FBD), showing all the forces acting on each separate body.

(2) Select proper coordinates for analysing the motion of each body.

(3) If there are any constraints, write the proper constraint eq^ns.

(4) Apply Newton's 2^nd law of motion :- $\vec{F} = m\vec{a}$ for each body. This leads to a system of eq^ns.

(5) Solve these eq^ns:-

(a) Identify the known and unknown quantities. Check that the no. of eq^ns equals the no. of unknowns

(b) Check the eq^ns using dimensional analysis

(c) After solving, check the final sol^n using back substitution.

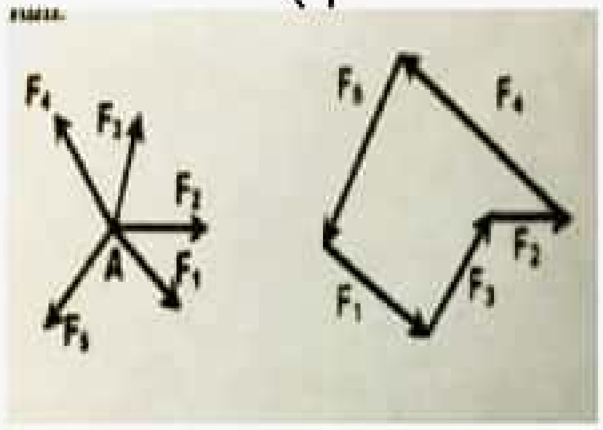
(6) If the velocity ($\vec{v}$) or position ($\vec{x}$) is required, proceed from a knowledge of acceleration ($\vec{a}$) as found from eq^ns in step (5) and apply kinematics

Eg. $\frac{d\vec{v}}{dt} = \vec{a}$ (known) and then integrate.

$\boxed{\Rightarrow \text{Equilibrium of Concurrent forces:-}}$

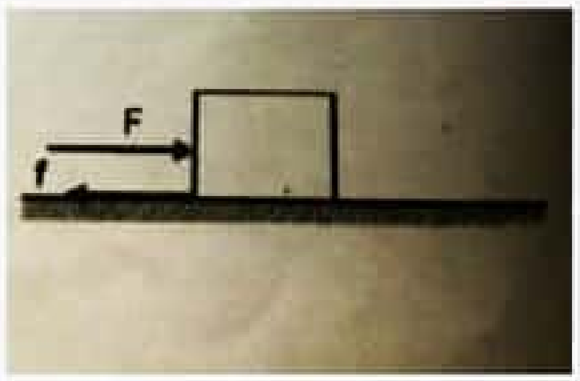
If the no. of forces that are acting on a particle can be taken along the sides of any polygon both in direction as well as in magnitude, it will be in equilibrium.

Suppose that the force F_1, F_2, F_3, F_4 and F_5 are acting on the particle A and if they are in equilibrium then they will form a pentagon

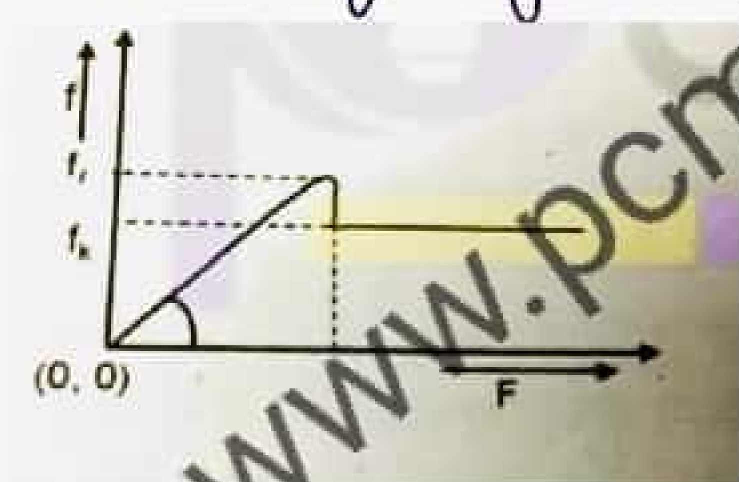


⇒ Frictional force:-

Frictional force comes into play between two surfaces whenever there is relative motion or a tendency of relative motion between two surfaces in contact. Frictional force has the tendency to stop relative motion between the surfaces in contact. Friction is a self-adjusting force. It changes its direction and magnitude acc. to the applied force or the forces, which causes a tendency in the body to move. If the force increases then the opposing force also increases until the body moves beyond which it remains constant. If the applied force is plotted against the frictional force we obtain a graph as shown.



The graph shows that first frictional force increases to a certain maximum value f_t with F and then suddenly decreases to a constant value f_k. For the range from 0 to f_t frictional force is equal and opposite to F and hence block does not move. In this range, friction force is static.



Thus, friction can be classified as:-

(a) Static friction:- It acts between surfaces in contact not in relative motion. It opposes the tendency of relative motion.

(b) Kinetic friction:- It acts between surfaces in contact which are in relative motion. It opposes the relative motion between the surfaces

⇒ Rolling friction:- When a body rolls on a rough surface the frictional force developed is known as rolling friction. It is generally less than the kinetic friction or limiting friction.

Laws of Static friction:-

Static friction, acting between the surfaces in contact, opposes the tendency of relative motion between the surfaces.

The frictional force acts tangentially along the surfaces in contact, and the maximum value (or limiting value) of this force is proportional to the normal reaction between the two surfaces. The force of friction between two bodies is an adjustable force, only its maximum or limiting value is proportional to the normal reaction. Secondly, the direction of this force is determined by all other forces acting on the body that is by the forces that tend to cause relative motion. The force of static friction acts in a direction so as to oppose the other forces that tend to cause relative motion between the surfaces in contact.

Now, $f_s(max) \propto N$ where $f_l = f_s(max) = f_s(max) = \mu_s N$

Here, μ_s = Co-efficient of static friction.

N = normal reaction of the block from the surface.

$$\boxed{0 \leq f_s \leq \mu N}$$

When F exceeds f_l block starts moving and frictional force decreases to a constant value f_k. f_k is called kinetic friction and it has unique value which is given by

$$\boxed{f_k = \mu_k N}$$

Here μ_k = Co-efficient of kinetic friction

N = Normal reaction

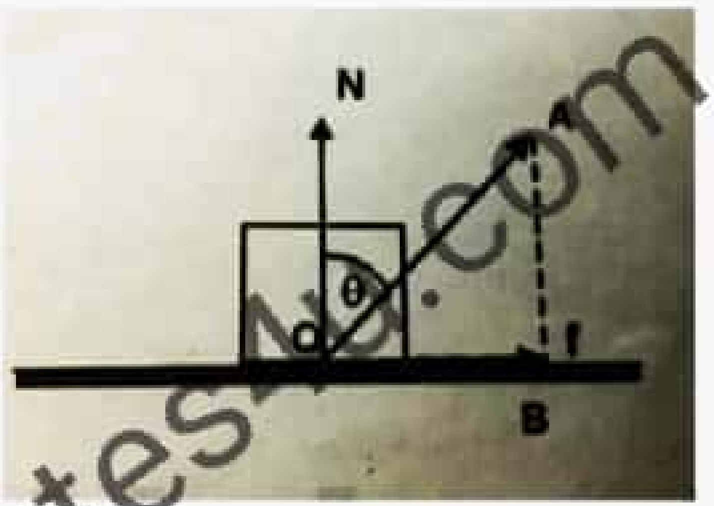
⇒ **Angle of friction:-** The angle made by the resultant reaction force with the vertical (normal reaction) in limiting condition i.e. when $f_s = f_e$, is known as the angle of the friction.

Now, in the triangle OAB,

$$\frac{AB}{OB} = \cot\theta$$

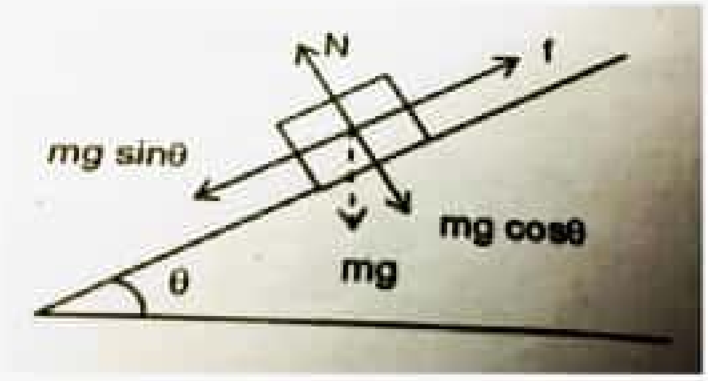
$$\Rightarrow OB = AB \tan\theta$$

or, $\tan\theta = \frac{f_l}{N}$



⇒ **Angle of Repose:-**

The angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide. Consider an inclined plane, whose inclination with horizontal is gradually increased, till the body placed on its surface just begins to slide down, then the angle made by the plane with horizontal is called angle of repose.



From the diag,

$$f = mg \sin\theta \quad \text{———} \quad (1)$$

$$N = Mg \cos\theta \quad \text{———} \quad (2)$$

Dividing (1) by (2)

$$\frac{f}{N} = \frac{Mg \sin\theta}{Mg \cos\theta} = \tan\theta$$

since $\frac{f}{N} = \mu$, $\tan\theta = \mu$

∴, coefficient of limiting friction is equal to the tangent to the angle of repose thus angle of repose is equal to the angle of friction.