CHAPTER - 3

MOTION IN A STRAIGHT LINE

Mechanics: Branch of Physics that deals with conditions of rest or motion of the material object around us.

Mechanics

Statics	Kinematics	Dynamics
It deals with study of object at rest or in equilibrium.	It deals with the study of motion without considering causes of motion.	It deals with study of motion of object. It also talks about cause of motion. i.e. force

Rest: An object is said to be at rest if it does not change its position with respect to time or surrounding.

Motion: If an object changes its position with respect to surrounding with time.

Point Object: If the position of an object changes by distance much greater than its own size in a reasonable duration of time, then the object may be considered as point object.

One dimensional: The motion of an object is said to be 1D, if only one out of 3 coordinates specifying the position of the object changes with time.
e.g. Motion of a train along a straight track.

Two dimensional: If 2 out of 3 coordinates specifying the position of the object change with time.
e.g. Car moving along zig-zag path.
Motion of planets around sun.

Three dimensional: Motion of an object is said to be 3D if all the three coordinates specifying the position of object change with time.
e.g. Kite flying on a windy day.
Motion of an aeroplane in space.

	Distance	Displacement
1.	It is defined as the actual length of path covered by a moving body.	It is defined as the shortest distance between the initial & final position of path covered by a moving body.
2.	It is a scalar quantity	It is a vector quantity.
3.	It can never be zero.	It can be zero.
4.	It is always +ve	It is +ve as well as -ve
5.	SI unit - m	SI unit - m

$\int(3t^2+2t+1)dt = \int dx$ ... $+c$ — constant of integration

$c = -2$

Speed: Distance travelled by a body per unit time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$x = t^3 + t^2 + t - 2$

$x = 4^3 + 4^2 + 4 - 2$

$= 64 + 16 + 4 - 2$

$= 82 m$

- SI unit = m/s
- Dimension = $[L T^{-1}]$
- It is a scalar quantity.
- It can not be zero.
- It is always +ve.

Average Speed: It is defined as the total path length travelled divided by total time interval during which the motion has taken place.

$$\text{Average speed} = V_{avg} = \frac{\text{Total path length}}{\text{Total time interval}}$$

Instantaneous Speed: Instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant.

$$v = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$$

Velocity: The rate of change of position of an object with time in a given direction is called its velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{dx}{dt}$$

- SI unit = m/s
- Dimension = $[L T^{-1}]$
- It is a vector quantity.
- It can be zero.
- It can be +ve as well as -ve.

Q) $x = 4t^2 + 3t + 2$

$t = 2$ sec

find v?

$v = \frac{dx}{dt} = \frac{d(4t^2+3t+2)}{dt}$

$= 8t + 3$

$= 8(2) + 3 \quad (\because t = 2)$

$= 19$ m/s

Average Velocity : It is defined as the displacement (Δx) divided by the time intervals in which displacement occurs:

$$V_{avg} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Instantaneous Velocity: It is defined as the limit of average velocity as the time interval Δt becom infinitely small.

$$V_{Ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Uniform Motion: If a body covers equal distance in equal intervals of time, no matter how small these intervals may be.

$$V_{Instantaneous} = V_{average}$$

e.g. Train travelling at 80km/h for 2h constantly.

Non-Uniform Motion: If a body covers unequal distance in equal intervals of time or vice-versa.
e.g. Kite flying on a windy day.

Acceleration: Rate of change of velocity.

$$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

- SI unit = m/s^2
- Dimension = $[L T^{-2}]$

Instantaneous Acceleration: The acceleration at an instant is the slope of the tangent to the v-t curve at that instant.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Positive Acceleration: If velocity of an object increases with time, its acceleration is +ve.
e.g. When a bus leaves a bus stop.

Negative Acceleration: If velocity of an object decreases with time, its acceleration is -ve.
It is also known as retardation or deceleration.
e.g. When we apply brakes.

Uniform Acceleration: It is defined as the change of equal velocity in equal intervals of time.

Non-uniform Acceleration: It is defined as the change of non-equal velocity in equal intervals of time.

Average Acceleration: It is the final velocity minus initial velocity per unit time taken.

Equations of Motion

Q) The displacement of a particle is given by $x = (t-2)^3$ where x is in metres & t in seconds. The distance covered by the particle in 4 seconds is

Ans ⇒ $x = (t-2)^2$

$= (4-2)^2$

$= 4$

$4m$

1- Ist equation: $v = u + at$

Derivation

$$a = \frac{\text{Change in Velocity}}{\text{Change in Time}}$$

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$$v = u + at$$

2. IInd equation: $s = ut + \frac{1}{2}at^2$

Derivation

$$V_{avg} = \frac{\text{Displacement}}{\text{Time}}$$

Displacement $= V_{avg} \times$ Time

$$s = \frac{u+v}{2} \times t$$

$$s = \frac{u+u+at}{2} \times t$$

$$s = \frac{2u+at}{2} \times t$$

$$s = \frac{2ut + at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$

3. IIIrd equation: $v^2 = u^2 + 2as$

Derivation

$$v = u + at$$

$$t = \frac{v-u}{a} \quad -(1)$$

Also,

$$s = ut + \frac{1}{2}at^2$$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$s = \frac{uv - u^2}{a} + \frac{1}{2a}\left(v^2 + u^2 - 2uv\right)$$

$$s = \frac{\cancel{2uv} - 2u^2 + v^2 + u^2 - \cancel{2uv}}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

Distance Covered in n^{th} second

$$a_n = S_n - S_{n-1}$$

$$S_{nth} = \underbrace{S_n}_{\text{Distance travelled in } n^{th} \text{ second}} - \underbrace{S_{n-1}}_{\text{Distance covered in } (n-1)^{th} \text{ second}}$$

$$\boxed{S_{6th} = S_6 - S_5}$$

$$S = ut + \frac{1}{2}at^2 \quad \text{[By } 2^{nd} \text{ equation of motion]}$$

$$S_{(n-1)} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$S_n = un + \frac{1}{2}an^2$$

$$S_{nth} = \left(un + \frac{1}{2}an^2\right) - \left\{u(n-1) + \frac{1}{2}a(n-1)^2\right\}$$

$$= \left(un + \frac{1}{2}an^2\right) - \left\{un - u + \frac{1}{2}a(n^2 - 2n + 1)\right\}$$

$$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{1}{2}an^2} + an - \frac{a}{2}$$

$$= u + an - \frac{a}{2}$$

$$\boxed{S_{nth} = u + \frac{a}{2}\left[2n - 1\right]}$$

Integration Method

(1) $a = \frac{dv}{dt}$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$dv = a\,dt$

$\int t^1 dt = \frac{t^{1+1}}{1+1} = \frac{t^2}{2}$

Now, Integrating it,

$$\int_u^v dv = a\int_0^t dt$$

$$[v]_u^v = a[t]_0^t$$

$$[v - u] = a[t - 0]$$

$$v - u = at$$

$$\boxed{v = u + at}$$

(2) $v = \frac{ds}{dt}$

$$ds = v\,dt$$

$$\int_0^s ds = \int (u + at)\,dt$$

$$[s - 0] = \int_0^t u\,dt + \int_0^t at\,dt$$

$$s = u[t - 0] + a\left[\frac{t^2}{2}\right]_0^t$$

$$s = ut + \frac{1}{2}at^2$$

Q) If $v = 3$
find acc

If velocity
with time
$v = 3t^2 + 5$
acceleration
Ans Accelera

(3)- $a = \frac{dv}{dt}$

$$a = \frac{dv}{dt} \times \frac{ds}{ds}$$

$$a = \frac{ds}{dt} \times \frac{dv}{ds}$$

$$a = v \times \frac{dv}{ds}$$

$$a = \frac{vdv}{ds}$$

(as differentiat
& for $x^n = n$
Now, Accele
Acceleration

$$ads = vdv$$

$$a\int_0^s ds = \int_u^v vdv$$

$$a(s-0) = \left[\frac{v^2}{2}\right]_u^v$$

$$2as = (v^2 - u^2)$$

$$\boxed{v^2 = u^2 + 2as}$$

$$v = \frac{ds}{dt}$$

$$ds = vdt$$

$t = (n-1)$ $\quad S = S_{n-1}$

$t = (n)$ $\quad S = S_n$

$$\int_{S_{n-1}}^{S_n} ds = (u+at)dt$$

$$[S_n - S_{n-1}] = udt + atdt$$

$$S_{nth} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt$$

$$S_{nth} = u[n-(n-1)] + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

NOTE:

1) If a body falls vertically downward, then acceleration due to gravity $= +g$.

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$

2) If a body moves vertically upward, the acceleration due to gravity $= -g$.

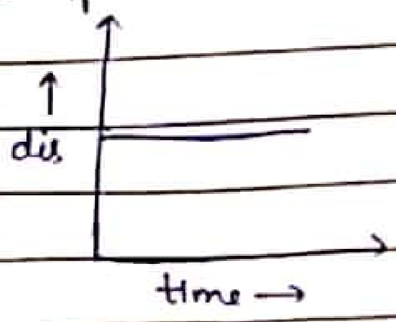
$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

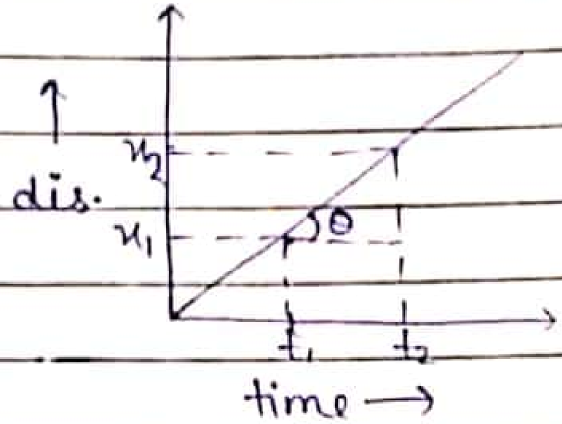
Graphs

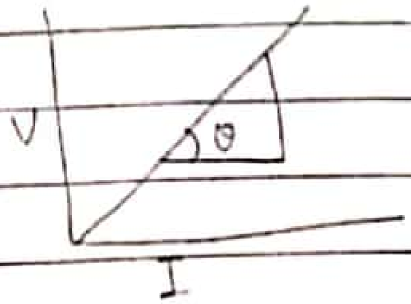
1)



Body at rest

2)





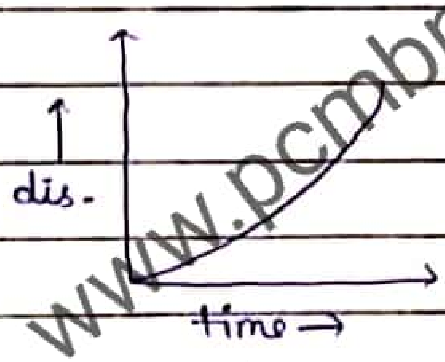
V-I graphs give

<u>Body moving with uniform velocity</u>

$$\text{Slope } \tan\theta = \frac{p}{b} = \frac{x_2 - x_1}{t_2 - t_1}$$

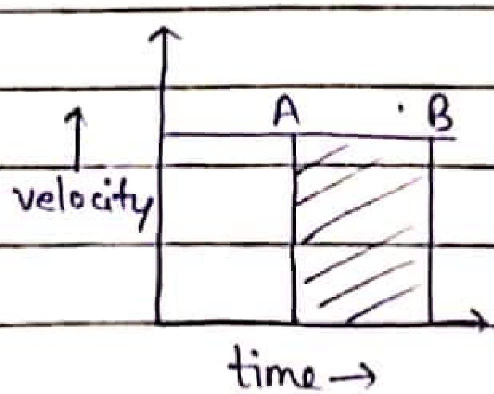
☆ <u>Slope of distance-time give speed of object</u>

3)



<u>Body moving with constant acceleration</u>

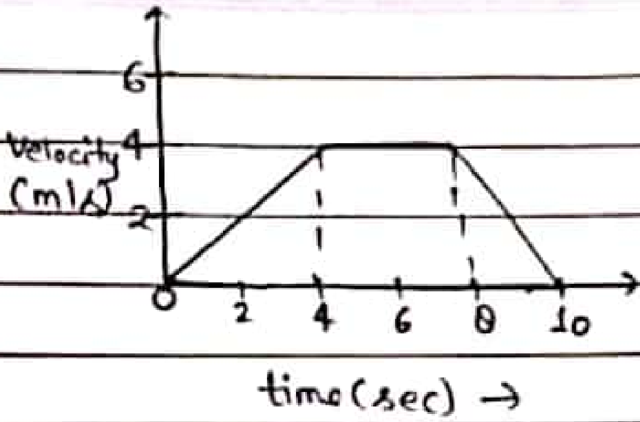
4)



<u>Velocity-time graph for uniform motion</u>

Area = displacement = Velocity × Time

Q)



(i) Calculate total displacement.

Ans - Displacement $= \frac{1}{2}\times 4\times \cancel{4}^{2} + 4\times 4 + \frac{1}{2}\times 2\times 4$

$= 8 + 16 + 4$

$= 28\,m$

(ii) Calculate acceleration from $t=0$ to $t=4$ sec.

Ans $a = \frac{4-0}{4} = \frac{4}{4} = 1\,m/s^2$

(iii) Calculate acceleration from $t=4$ to $t=8$ sec.

Ans $0\,m/s^2$

(iv) Calculate acceleration from $t=10$ to $t=8$ sec.

Ans $a = \frac{0-4}{2} = \frac{-4}{2} = -2\,m/s^2$.

Relative Velocity

The relative velocity of an object 2 with respect to 1 when both are in motion is the time rate of change of position of object 2 with respect to 1.

Let $x_1(0)$ & $x_2(0)$ are position coordinates at time t=

At time = t,

$x_1(t) = x_1 0 + v_1 t$ - (1)

$x_2(t) = x_2 0 + v_2 t$ - (2)

(2) - (1)

$x_2 t - x_1 t = x_2 0 - x_1 0 + [v_2 - v_1] t$

$(x_2 t - x_2 0) = (x_1 t - x_1 0) + (v_2 - v_1) t$

Displacement of object 2 in time t. Displacement of object 1 in time t.

$\frac{\text{Relative displacement of 2 w.r.t 1}}{\text{Time}} = v_2 - u_1$

Relative velocity of 2 wrt 1

$v_{21} = v_2 - v_1$

Relative velocity of 1 wrt 2

$v_{12} = v_1 - v_2$

NOTE:

- When two bodies move in opposite direction, then in order to find relative velocity, we have to add the speed of one body to the other

eg. $\xrightarrow{50 \text{ km/h}}$ Bus $\xleftarrow{30 \text{ km/h}}$ Boy

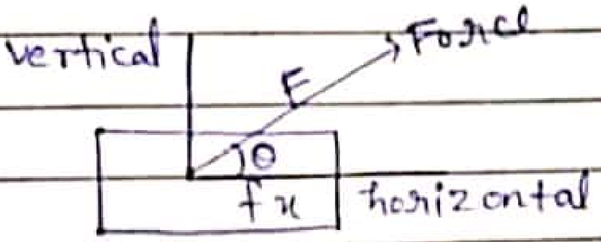
Relative Velocity = 50 + 30 = 80 km/h

- When two bodies move in same direction, then in order to find relative velocity, we have to subtract the speed of one body from another.

e.g. Bus $\xrightarrow{50\,km/h}$

Boy $\xrightarrow{20\,km/h}$

Relative Velocity $= 50 - 20 = 30\,km/h$.



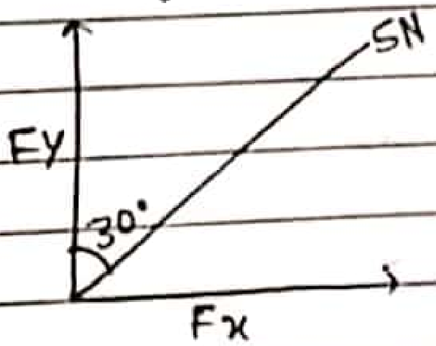
↳ break down of component

$$\cos\theta = \frac{b}{h}$$

$$\cos\theta = \frac{f_x}{F}$$

$$f_x = F\cos\theta$$

Q)



(i) Calculate F_x

(ii) Calculate F_Y

(i) $f_x = 5\sin 30°$
$= 5 \times \frac{1}{2}$
$= 2.5N$

(ii) $F_Y = 5\cos 30°$
$= 5 \times \frac{\sqrt{3}}{2}$
$= \frac{5\sqrt{3}}{2}$